Different Measures of Average Treatment Effect for Binary Outcome, Estimating by Propensity Scoring

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Questions ??

- **How to determine whether a treatment/risk factor is <u>on average</u> effective in reducing/increasing disease risk, in a large observational study with a lot of observed confounders ?**

- **Z**: <u>**binary**</u> treatment indicator.
- **Y**: <u>**binary**</u> outcome.
- **X**: covariate.
- $\mathbf{Y_i^{(0,1)}}$: potential outcome
- Population average risk:

$$P^{(1)} = \frac{1}{N}\sum_{i=1}^{N} Pr(Y_i^{(1)} = 1)$$

$$P^{(0)} = \frac{1}{N}\sum_{i=1}^{N} Pr(Y_i^{(0)} = 1)$$

<u>Example</u>

- Smoking status
- Lung Cancer
- Age
- Potential cancer status if I … or not ..
- Population average cancer risk

Propensity Scoring (P.S.)

- Define: $\mathbf{e_T(X) = P_r(Z=1| X)}$
  (e.g. the risk of being smoking at giving age.)

- $e_T(x)$ is a balancing score → $\mathbf{X \perp\!\!\!\perp Z \mid e_T(x)}$.
  (e.g. for people with same $e_T(X)$, the distribibuion of age is same across smoking groups.)

- **Typical tool for studying causal inference.**
  - The marginal inference of Z to Y (average over X).

- **Two conditions for valid causal inference:**
  1). Treatment assignment is strongly ignorable
  2). Close to correctly specified: Z relationship to X.

P.S. Procedure

1. Estimate e(x).
2. Take subjects with overlapped $\widehat{e(x)}$ after ordering.
3. Subclassification of $\widehat{e(x)}$ into bins.
4. If $\mathbf{f}_{\mathbf{X|Z=1,\ jth\ bin}} \approx \mathbf{f}_{\mathbf{X|Z=0,\ jth\ bin,}}$ hold for **all Xs** within **all subclasses,** then move on**; o.w, back to step-1.**

5. $\widehat{P_j^{(1)}} = average(Y|Z=1, j^{th} \text{ bin})$ $\qquad \widehat{P^{(1)}} = \sum_{j=1}^{J} \omega_j \widehat{P_j^{(1)}}$

$\widehat{P_j^{(0)}} = average(Y|Z=0, j^{th} \text{ bin})$ $\qquad \widehat{P^{(0)}} = \sum_{j=1}^{J} \omega_j \widehat{P_j^{(0)}}$

6. **Choose** measure of average treatment effect, and estimate it.

Collapsibility

- **Collapsible:**

$$\sum_{j=1}^{J} w_j f(P_k^{(1)}, P_k^{(0)}) = f(\sum_{j=1}^{J} w_j P_k^{(1)}, \sum_{j=1}^{J} w_j P_k^{(0)})$$

- **A characteristic of the <u>chosen measure.</u>**
- **Not depends on model**.
- **Three Types**

Hypothetical example: Perfect Randomized Trial

	Z=1	Z=0	Size
Age<65	$P_1(Y^{(1)}=1)=0.4$	$P_1(Y^{(0)}=1)=0.2$	1000
Age⩾65	$P_2(Y^{(1)}=1)=0.8$	$P_2(Y^{(0)}=1)=0.6$	1000
	$P(Y^{(1)}=1)=0.6$	$P(Y^{(0)}=1)=0.4$	2000

Young: $OR_1 = \frac{0.4/(1-0.4)}{0.2/(1-0.2)} = 2.67$

Old: $OR_2 = \frac{0.8/(1-0.8)}{0.6/(1-0.6)} = 2.67$

Marginal: $OR = \frac{0.6/(1-0.6)}{0.4/(1-0.4)} = 2.25 \neq \frac{1}{2}2.67 + \frac{1}{2}2.67$

1. **Collapsible**
2. **Collapsible under assumptions**
3. **Not Collapsible.**

Example from: Greenland, 1999, also see: 2002, 1987, 1985, ; Whittaker, 1990

$P^{(1)}$-$P^{(0)}$ – Average Risk Difference

$$P_j^{(1)} = Pr(Y^{(1)} = 1 | j^{th} \text{ bin}) = \frac{1}{N_j} \sum_{i=1}^{N_j} Pr(Y_i^{(1)} = 1)$$

$$P_j^{(0)} = Pr(Y^{(0)} = 1 | j^{th} \text{ bin}) = \frac{1}{N_j} \sum_{i=1}^{N_j} Pr(Y_i^{(0)} = 1)$$

$$P^{(1)} - P^{(0)} = \sum_{j=1}^{J} \omega_j P_j^{(1)} - \sum_{j=1}^{J} \omega_j P_j^{(0)} = \sum_{j=1}^{J} \omega_j (P_j^{(1)} - P_j^{(0)})$$

$$= \frac{1}{N} \sum_{i=1}^{N} [Pr(Y_i^{(1)} = 1) - Pr(Y_i^{(0)} = 1)] \text{ , } \omega_j = \frac{N_j}{N}$$

Collapsible: overall ← bin-specific ← individual level

- The difference of average risk
- A weighted average of bin-specific treatment effect.
- The average of individual risk difference.

$P^{(1)}/P^{(0)}$ – Marginal Relative Risk

- **Not Collapsible, in general.**

$$\frac{P^{(1)}}{P^{(0)}} = \frac{\sum_{j=1}^{J} \omega_j P_j^{(1)}}{\sum_{j=1}^{J} \omega_j P_j^{(0)}} \neq \sum_{j=1}^{J} \omega_j \frac{P_j^{(1)}}{P_j^{(0)}}$$

- **Collapsible, w/ constant treatment effect assumption.**

$$\text{If } \frac{Pr(Y_i^{(1)} = 1)}{Pr(Y_i^{(0)} = 1)} = r \quad for \quad i = 1, 2, \ldots N \text{ , then}$$

$$\frac{P^{(1)}}{P^{(0)}} = \sum_{j=1}^{J} \omega_j \frac{P_j^{(1)}}{P_j^{(0)}} = r$$

P.S.: $\frac{P^{(1)}/(1-P^{(1)})}{P^{(0)}/(1-P^{(0)})}$ – Marginal Odds Ratio (OR)

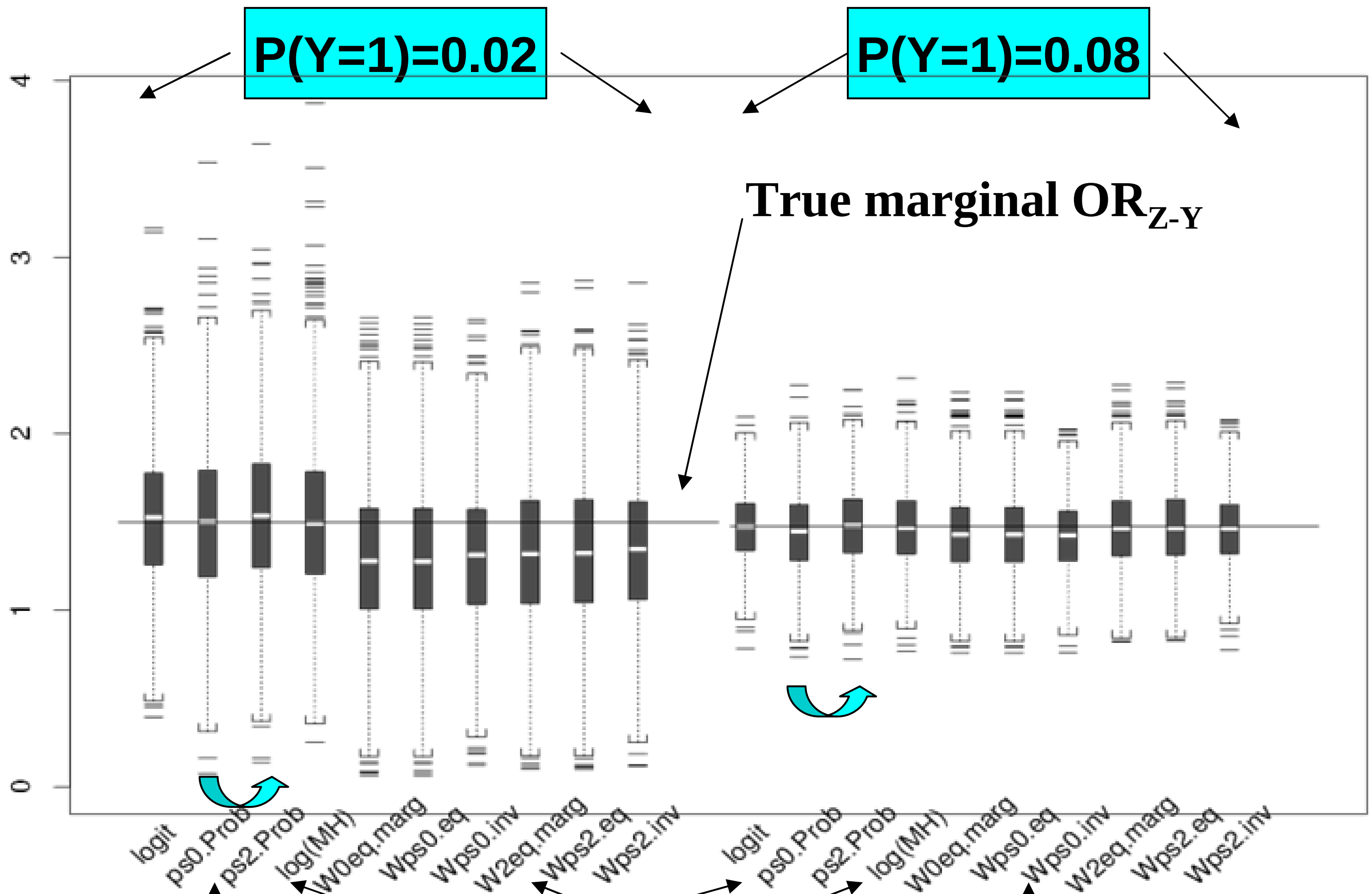
- **Not collapsible:**
  - individual level → bin-specific level
  - bin-specific level → overall effect level

w/ or w/o constant treatment effect assumption

$$\frac{P^{(1)}/(1-P^{(1)})}{P^{(0)}/(1-P^{(0)})} = \frac{\sum_{j=1}^{J}\omega_j P_j^{(1)}/(1-\sum_{j=1}^{J}\omega_j P_j^{(1)})}{\sum_{j=1}^{J}\omega_j P_j^{(0)}/(1-\sum_{j=1}^{J}\omega_j P_j^{(0)})}$$

$$\neq \sum_{j=1}^{J}\omega_j \frac{P_j^{(1)}/(1-P_j^{(1)})}{P_j^{(0)}/(1-P_j^{(0)})} \neq \frac{\sum_{j=1}^{J}\omega_j P_j^{(1)}/(1-P_j^{(1)})}{\sum_{j=1}^{J}\omega_j P_j^{(0)}/(1-P_j^{(0)})}$$

Log(Marginal $OR_{Z\text{-}Y}$), + constant treatment effect



$$log(\frac{P^{(1)}/(1-P^{(1)})}{P^{(0)}/(1-P^{(0)})})$$

M-H

$$\sum_{j=1}^{J} \omega_j log(\frac{P_j^{(1)}/(1-P_j^{(1)})}{P_j^{(0)}/(1-P_j^{(0)})})$$

Methods lable

"logit": logistic reg

"ps0.Prob": P.S.using marginal probability **w/o** check balance

"ps2.Prob": same P.S, w/ check balance up to 2 moments

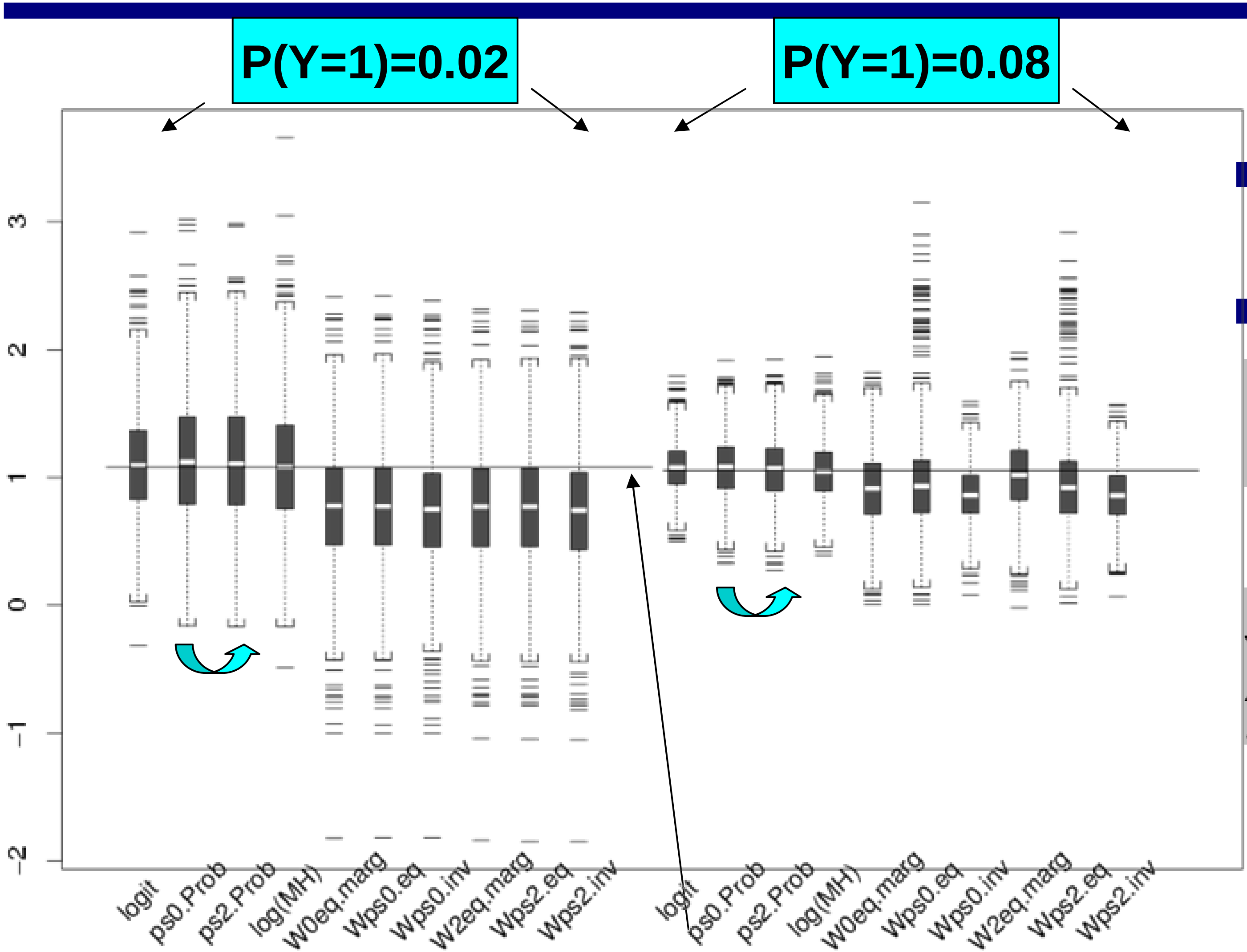
"Log(MH)": M-H estimator

Other: $\sum_{j=1}^{J} \omega_j \hat{\beta}_j$

"inv": inverse variance weights

"equal": equal weights

Log(Marginal $OR_{Z\text{-}Y}$), No constant treatment effect



True marginal $OR_{Z\text{-}Y}$

- **Same methods label**
- **Further apart:**

$$log(\frac{P^{(1)}/(1-P^{(1)})}{P^{(0)}/(1-P^{(0)})})$$

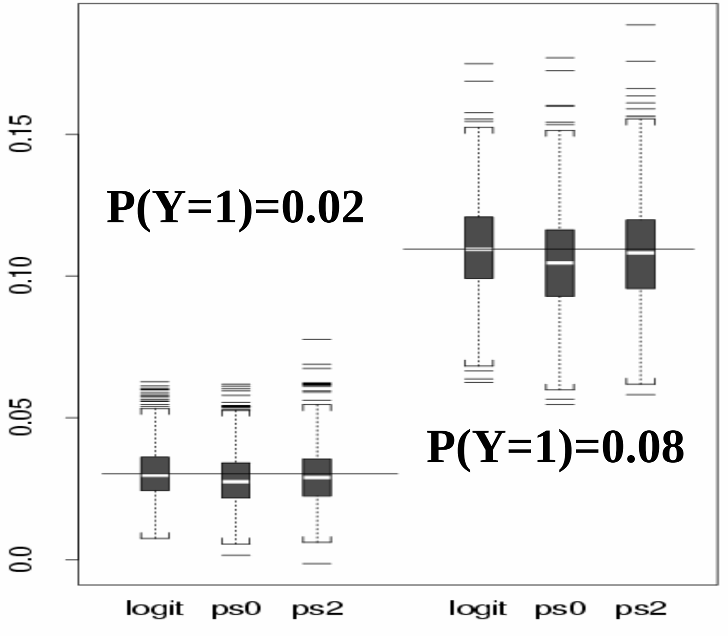
away from

$$\sum_{j=1}^{J}\omega_j log(\frac{P_j^{(1)}/(1-P_j^{(1)})}{P_j^{(0)}/(1-P_j^{(0)})})$$

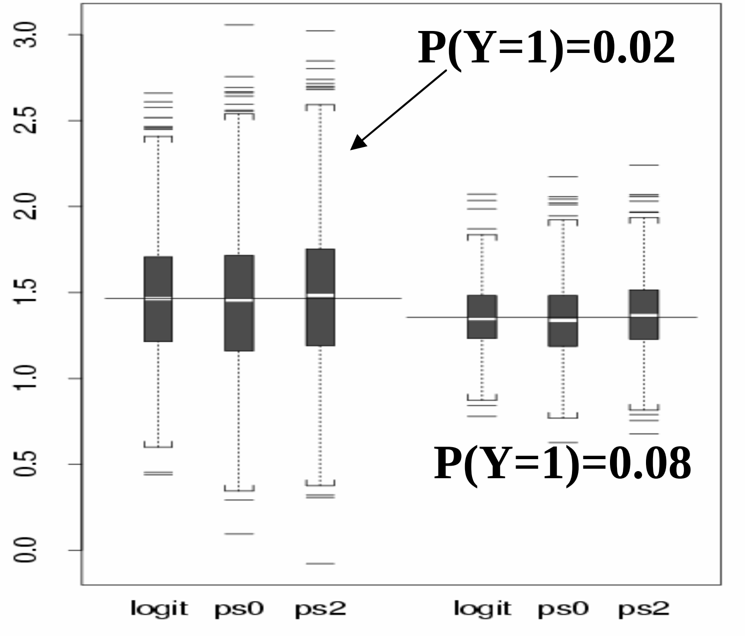
regardless P(Y=1).

Average Risk Difference & Marginal R.R.

$P^{(1)} - P^{(0)}$, $\log(OR_{Z\text{-}Y|X})=1.5$



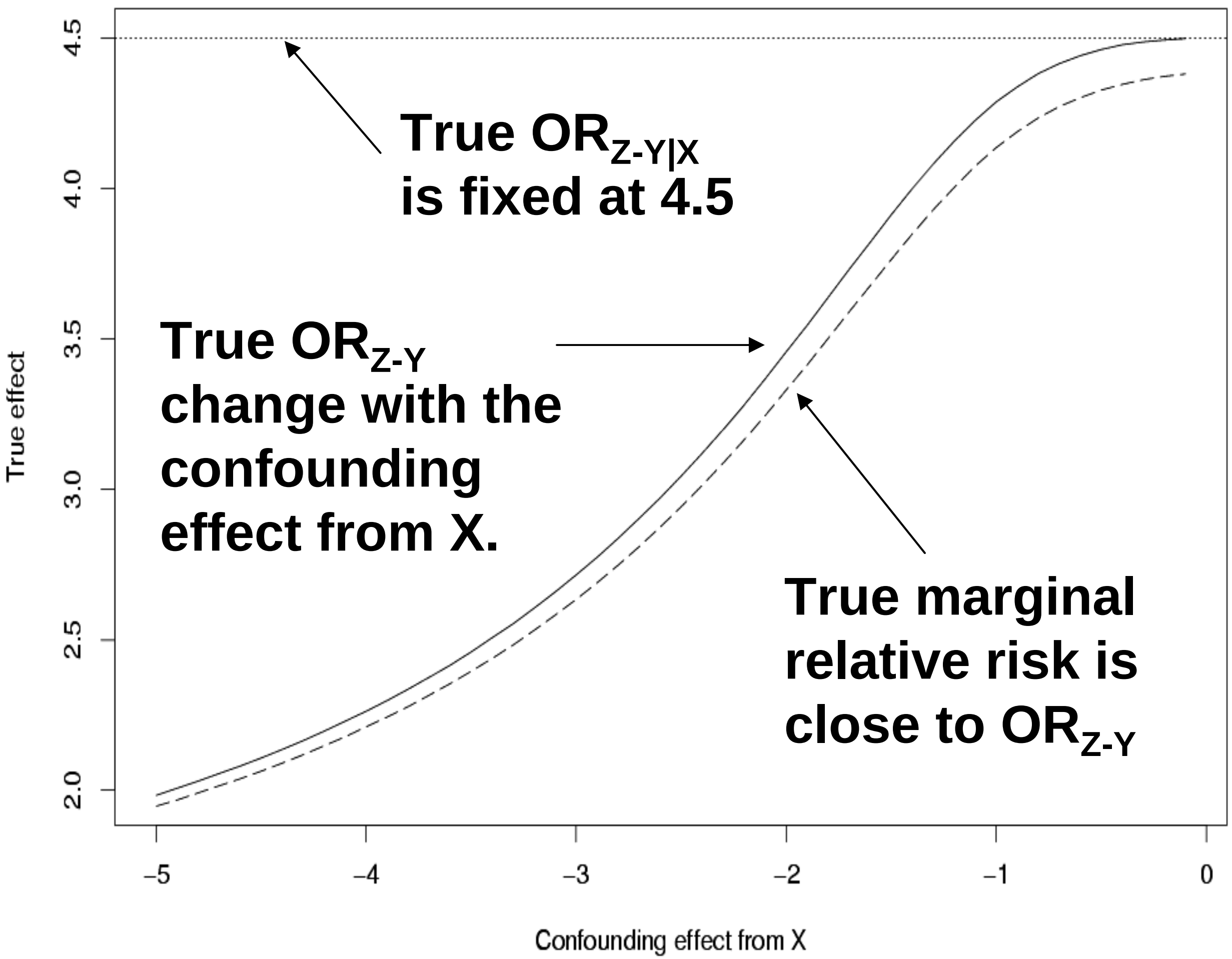
$\mathrm{Log}(P^{(1)}/P^{(0)})$, $\log(OR_{Z\text{-}Y|X})=1.5$



- Under rare disease, ARD is highly influenced by P(Y=1).
- Marginal RR estimated by P.S. performs nice.

$OR_{Z\text{-}Y|X} \neq OR_{Z\text{-}Y}$, even when disease is rare

Comparing $OR_{Z\text{-}Y|X}$ & $OR_{Z\text{-}Y}$



Setting:

$$X \sim N(0,1)$$

$$logit[E(Y|,X)] =$$

$$\beta_0 + log(4.5)Z + \beta_x X$$

$Pr(Y=1) = 0.02$,

by adjusting β_0

N = 8000

Summary

- **With constant treatment effect + the increasing of disease prevalence,** the performance on estimators of weighted average of bin-specific effect type become better. **Without** constant treatment effect, their performance is bad.
- With the **increasing of disease prevalence,** model performance for different treatment measures become better.
- P.S: It is not always correct to say – "average treament effect is a weighted average of bin-specific treatment effect".

  It really depends on your choice of treatment effect measure.
- In general, it is better to critically examine **which treatment effect measure is best for your problem** before applying technique to estimate it.

Thanks!

Questions?