MATH 152
Mrs. Bonny Tighe

QUIZ 9 A
25 points
12.9.12.10

NAME Answer

Section________ Wed 11/16/05

1. Find a power series representation for the function and determine the interval of convergence. $f(x) = \dfrac{1}{4-x^2} = \dfrac{1/4}{1-\left(\frac{x}{2}\right)^2} = \frac{1}{4}\sum_{n=0}^{\infty}\left(\left(\frac{x}{2}\right)^2\right)^n$

$$\frac{1}{4}\sum_{n=0}^{\infty}\frac{x^{2n}}{2^{2n}} = \boxed{\sum_{n=0}^{\infty}\frac{x^{2n}}{2^{2n+2}} \text{ or } \sum_{n=0}^{\infty}\frac{x^{2n}}{4^{n+1}}}$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{x^{2n+1}}{2^{2n+1}}\cdot\frac{2^{2n+2}}{x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{x}{4}\right| < 1$$

$R=4 \quad I=(-4,4)$

$-4<x<4$

$$(-4)\quad \sum_{n=0}^{\infty}\frac{(-4)^{2n}}{4^{n+1}} = (-1)^{2n}4^{n-1}$$

div

2. Find a power series representation for the function and determine the radius of convergence.

a) $f(x) = \ln(2-x)$

$$f'(x) = \frac{1}{2-x}(-1) = \frac{-1/2}{1-\left(\frac{x}{2}\right)}$$

$$f'(x) = -\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n = \sum_{n=0}^{\infty}\frac{-x^n}{2^{n+1}}$$

(integrate)

$$f(x) = \sum_{n=0}^{\infty}\frac{-x^{n+1}}{(2^{n+1})(n+1)}$$

$$\lim_{n\to\infty}\left|\frac{x^{n+2}}{2^{n+2}(n+2)}\cdot\frac{2^{n+1}(n+1)}{x^{n+1}}\right| =$$

$$\lim_{n\to\infty}\left|\frac{x}{2}\left(\frac{n+1}{n+2}\right)\right| = \left|\frac{x}{2}\right|<1$$

$R=2$

b) $f(x) = \dfrac{1}{(1+x)^2}$ $\quad \int f(x) = \dfrac{-1}{1+x}$

$$\sum_{n=0}^{\infty}-(-x)^n = \sum_{n=0}^{\infty}(-1)^{n+1}x^n$$

(differentiate) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}n\,x^{n-1}$

$$\lim_{n\to\infty}\left|\frac{(n+1)\,x^n}{n(x^{n-1})}\right| = |x|<1$$

$R=1$

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots$$

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

3. Find the Taylor series for f(x) centered at the given value of a.

a) $f(x)=\cancel{x^2}\ \frac{1}{x}$, $a=1$

b) $f(x)=\cancel{x}e^{2x}$, $a=1$

$f'(x)=-\frac{1}{x^2}$

$f''(x)=+\frac{2}{x^3}$

$f'''(x)=\frac{-3\cdot 2}{x^4}$

$$1+\frac{-1}{1!}(x-1)+\frac{2(x-1)^2}{2!}+\frac{-3\cdot 2}{3!}(x-1)^3+\frac{4\cdot 3\cdot 2(x-1)^4}{4!}+\dots$$

$$\boxed{1+\sum_{n=1}^{\infty}(-1)^n(x-1)^n}$$

$$x\sum_{n=0}^{\infty}\frac{(2x)+1)^n}{n!}$$

$$\boxed{\sum_{n=0}^{\infty}\frac{(2x+1)^n(x)}{n!}}$$

5. Find the Maclaurin series of f and its radius of convergence. $f(x)=\cancel{\frac{1}{4+x^3}}\ \sqrt{1+x}$

$$f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\dots$$

$$1+\frac{1/2}{1!}x+\frac{-1/4}{2!}x^2+\frac{3/8\ x^3}{3!}+\frac{-15/16\ x^4}{4!}+\dots$$

$$1+\frac{(-1)^{n+1}(1.3.5\cdots(2n-1)}{2^n n!\ x^n}=\boxed{1+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(1.3.5\dots(2n-1))x^n}{2^n n!}}$$

$f'(x)=\frac{1}{2}(1+x)^{-1/2}$

$f''(x)=-\frac{1}{4}(1+x)^{-3/2}$

$f'''(x)=\frac{3}{8}(1+x)^{-5/2}$

$f''''(x)=\frac{-15}{16}(1+x)^{-7/2}$

6. Evaluate the indefinite integral as an infinite series. $\int\sin(x^2)dx$

$$\int\sin(x^2)=\int\sum_{n=0}^{\infty}\frac{(-1)^n(x^2)^{2n+1}}{(2n+1)!}=\int\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+2}}{(2n+1)!}=$$

$$C+\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$$