MATH 152
Mrs. Bonny Tighe

QUIZ 8A

25 points
12.7, 12.8

NAME Answers

Section ________ Wed 11/9/05

Test each of the series for convergence or divergence.

State and use the Integral Test

a) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

Suppose f is a cont, pos, decreasing function on $[1,\infty)$ & $a_n = f(n)$ then if $\int_1^\infty f(x)dx$ is convergent then $\sum a_n$ is also.

$u = \ln n \quad du = \frac{1}{n}$

$dv = n^{-2} \quad v = -\frac{1}{n}$

$\lim_{t\to\infty} \int_1^t \ln n \frac{1}{n^2} dn$

$\lim_{t\to\infty} \ln n\left(-\frac{1}{n}\right) - \int -\frac{1}{n}\left(\frac{1}{n}\right)dn =$

$\lim_{t\to\infty} \left(-\frac{\ln n}{n} + -\frac{1}{n}\right|_1^t = \lim_{t\to\infty}\left(\frac{\ln t}{t} - \frac{t}{t}\right) - (-0-1)$

$= 0 - 0 + 0 + 1$

$= 1$ So convergent

State and use the Ratio Test

b) $\sum_{n=1}^{\infty} \frac{5^n}{n!(n+1)}$

If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$ then
$L<1$ convergent
$L=1$ inconclusive
$L>1$ divergent

$\lim_{n\to\infty} \left|\frac{5^{n+1}}{(n+1)!(n+2)} \cdot \frac{n!(n+1)}{5^n}\right|$

$\lim_{n\to\infty} \left|\frac{5}{n+1} \cdot \frac{n+1}{n+2}\right| = 0 < 1$

So convergent absolutely

State and use the Comparison Test.

c) $\sum_{n=2}^{\infty} \frac{\cos \pi n}{n^2+n}$

$\cos \pi n = \pm 1$ or $(-1)^n$

Suppose $\sum a_n$ + $\sum b_n$ are series with positive terms i) If $\sum b_n$ is convergent and $a_n \le b_n$ for all n, then so is $\sum a_n$ ii) If $\sum b_n$ is divergent and $a_n \ge b_n$ for all n, then so is $\sum a_n$.

$b_n = \frac{1}{n^2}$ which is convergent p-series

and $a_n = \frac{(-1)^n}{n^2+n} \le \frac{1}{n^2}$ for all n

then $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+n}$ is also convergent

State and use the Alternating Series Test

d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+2}}$

If:
i) $b_{n+1} \le b_n$

$\frac{1}{\sqrt{n+3}} \le \frac{1}{\sqrt{n+2}}$ ✓ yes

and ii) $\lim_{n\to\infty} b_n = 0$ then

$\lim_{n\to\infty} \frac{1}{\sqrt{n+2}} = 0$ yes

then $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ is convergent

So $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+2}}$ is convergent

2. Find the radius of convergence and the interval of convergence for each of the given power series.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n \ln n}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)\ln(n+1)} \cdot \frac{n \ln n}{x^n} \right| =$

$\lim_{n\to\infty} \left| x \cdot \frac{n}{n+1} \cdot \frac{\ln n}{\ln(n+1)} \right| = |x| < 1$

$R = 1$ $\quad I = [-1, 1]$

$\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{n \ln n}$ ✓ Convergent

$\sum_{n=0}^{\infty} \frac{(-1)^n(1)^n}{n \ln n}$ Convergent

b) $\sum_{n=0}^{\infty} \frac{(\frac{x}{3})^n}{n!}$

$\lim_{n\to\infty} \left| \frac{(\frac{x}{3})^{n+1}}{(n+1)!} \cdot \frac{n!}{(x/3)^n} \right| =$

$\lim_{n\to\infty} \left| \frac{x/3}{n+1} \right| = 0 < 1$ always true so

$R = \infty$ $\quad I = (-\infty, \infty)$

c) $\sum_{n=0}^{\infty} \frac{(2x+1)^n}{n+1}$

$\lim_{n\to\infty} \left| \frac{(2x+1)^{n+1}}{n+2} \cdot \frac{n+1}{(2x+1)^n} \right|$

$\lim_{n\to\infty} \left| 2x+1 \left(\frac{n+1}{n+2}\right) \right| =$

~~$|2x+1| < 1$~~

$-1 < 2x+1 < 1$

$-2 < 2x < 0$

$-1 < x < 0$

$R = 1/2$ $\quad I = [-1, 0)$

$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ ~~dvg~~ Cvg AST

$\sum_{n=0}^{\infty} \frac{1^n}{n+1}$ dvg

d) $\sum_{n=0}^{\infty} \frac{3^n (x+2)^n}{1 \cdot 4 \cdot 7 \cdot 10 \cdots (3n-2)}$

$\lim_{n\to\infty} \left| \frac{3^{n+1}(x+2)^{n+1}}{1 \cdot 4 \cdot 7 \cdots (3n-2)(3n+1)} \cdot \frac{1 \cdot 4 \cdot 7 \cdots (3n-2)}{3^n (x+2)^n} \right|$

$\lim_{n\to\infty} \left| \frac{3(x+2)}{3n+1} \right| = 0 < 1$ always so

$R = \infty$ $\quad I = (-\infty, \infty)$