MATH 152
Mrs. Bonny Tighe

QUIZ 8

25 points
12.7, 12.8

NAME Answers

Section ________ Wed 11/9/05

Test each of the series for convergence or divergence:

State and use the Comparison Test.

a) $\sum_{n=2}^{\infty} \frac{\cos \pi n}{n^3+n}$ $\quad \cos \pi n = \pm 1$

$\sum \frac{1}{n^3}$ is convergent p-series $p>1$

$\frac{1}{n^3+n} \leq \frac{1}{n^3}$ for all n So

$\sum_{n=2}^{\infty} \frac{\cos \pi n}{n^3+n}$ is absolutely convergent

$a_n = \frac{(-1)^n}{n^3+n} \quad b_n = \frac{1}{n^3}$

If $a_n \leq b_n$ for all n then a_n is also convergent

State and use the Alternating Series Test

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{5^n}$

If i) $b_{n+1} \leq b_n$ for all n and ii) $\lim_{n\to\infty} b_n = 0$ then $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ is convergent.

i ✓ $\frac{1}{5^{n+1}} \leq \frac{1}{5^n}$ $\quad$ ii $\lim_{n\to\infty} \frac{1}{5^n} = 0$ ✓

So convergent.

State and use the Integral Test

c) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ positive decreasing continuous

$a_n = \frac{\ln n}{n^2} = f(n)$

If $\int_1^{\infty} f(x)dx$ is convergent then $\sum a_n$ is also convergent

$\lim_{t\to\infty} \int_1^t \ln x \cdot \frac{1}{x} \cdot \frac{1}{x} dx$ $\quad u = \ln x \quad du = \frac{1}{x}dx \quad dv = x^{-2} \quad v = \frac{1}{-1}x^{-1}$

$\lim_{t\to\infty} \int_1^t \frac{\ln x}{x^2} dx = \ln x\left(-\frac{1}{x}\right) - \int \frac{1}{x^2}\left(\frac{1}{x}\right)dx$

$\lim_{t\to\infty} -\frac{\ln x}{x} + \frac{1}{2x^2}\Big|_1^t = \left(-\frac{\ln t}{t} + \frac{1}{2t^2}\right) - \left(\frac{\ln(1)}{1} + \frac{1}{2(1^2)}\right)$

$0 + 0 + 0 - \frac{1}{2} = -\frac{1}{2}$

Convergent

State and use the Ratio Test

d) $\sum_{n=1}^{\infty} \frac{10^n}{n!}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$ If $\begin{cases} L<1 & \text{cvg} \\ L=1 & \text{inconclusive} \\ L>1 & \text{divergent} \end{cases}$

$\lim_{n\to\infty} \left|\frac{10^{n+1}}{(n+1)!} \cdot \frac{n!}{10^n}\right| =$

$\lim_{n\to\infty} \left|\frac{10}{n+1}\right| = 0 < 1$

So absolutely convergent

Ratio Test $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$ Converges

2. Find the radius of convergence and the interval of convergence for each of the given power series.

a) $\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^n}{n+1}$

$\lim_{n\to\infty}\left|\frac{(x-1)^{n+1}}{n+2}\cdot\frac{n+1}{(x-1)^n}\right| =$

$\lim_{n\to\infty}\left|x-1\left(\frac{n+1}{n+2}\right)\right| = |x-1| < 1$

$I = (0, 2]$ $R = 1$

$-1 < x-1 < 1$

$0 < x < 2$

Test 0 $\sum_{n=0}^{\infty}\frac{(-1)^n(-1)^n}{n+1}$ Dvg ✓

Test 2 $\sum_{n=0}^{\infty}\frac{(-1)^n 1^n}{n+1}$ Cvg AST ✓

b) $\sum_{n=0}^{\infty}\frac{\left(\frac{x}{2}\right)^n}{n!} = \frac{x^n}{2^n n!}$

$\lim_{n\to\infty}\left|\frac{x^{n+1}}{2^{n+1}(n+1)!}\cdot\frac{2^n n!}{x^n}\right|$

$\lim_{n\to\infty}\left|\frac{x}{2(n+1)}\right| = 0 < 1$ always so

$R = \infty$ $I = (-\infty, \infty)$

c) $\sum_{n=0}^{\infty}\sqrt{n}(3x-1)^n$

$\lim_{n\to\infty}\left|\frac{\sqrt{n+1}\,(3x-1)^{n+1}}{\sqrt{n}\,(3x-1)^n}\right| =$

$\lim_{n\to\infty}\left|\sqrt{\frac{n+1}{n}}\,(3x-1)\right| = |3x-1| < 1$

$R = 1/3$ $I = (0, 2/3)$

$-1 < 3x-1 < 1$ $0 < 3x < 2$ $0 < x < 2/3$

test 0 $\sum_{n=0}^{\infty}\sqrt{n}(-1)^n$ Dvg ✓

test 2/3 $\sum_{n=0}^{\infty}\sqrt{n}(1)^n$ Dvg ✓

d) $\sum_{n=0}^{\infty}\frac{2^n(x-3)^n}{3\cdot5\cdot7\cdot9\cdots(2n+1)}$

$\lim_{n\to\infty}\left|\frac{2^{n+1}(x-3)^{n+1}}{3\cdot5\cdot7\cdots(2n+1)(2n+3)}\cdot\frac{3\cdot5\cdot7\cdot9\cdots(2n+1)}{2^n(x-3)^n}\right|$

$\lim_{n\to\infty}\left|\frac{2(x-3)}{2n+3}\right| = 0 < 1$ always so

$R = \infty$

$I: (-\infty, \infty)$