ORANGE 3pm 1st

MATH 152
Mrs. Bonny Tighe

QUIZ 7A
25 points
12.4,12.5, 12.6

NAME Answers
Wed. 11/2/05

Comparison
Ratio
Root
Integral

1. Test the series for convergence or divergence. State the test you use and show all work. If the series is an Alternating Series, find if it is Absolutely or Conditionally convergent. ~~Use each test at least one time.~~ AST

a) $\sum_{n=1}^{\infty}\left(\frac{-4n}{1+5n}\right)^n$ $\frac{(-1)^n 4^n \cancel{n^n}}{\cancel{(1+5n)^n}}$

$\boxed{\text{Root Test}}$

$\lim_{n\to\infty}\left|\frac{4n}{1+5n}\right| = 4/5 < 1$

so convergent

absolutely convergent

b) $\sum_{n=1}^{\infty}\frac{5^n}{n!}$ $\boxed{\text{Ratio Test}}$

$\lim_{n\to\infty}\left|\frac{5^{n+1}}{(n+1)!}\cdot\frac{n!}{5^n}\right| =$

$\lim_{n\to\infty}\left|\frac{5}{n+1}\right| = 0 < 1$ so

$\boxed{\text{absolutely convergent}}$

c) $\sum_{n=2}^{\infty}\frac{(-1)^n(n-1)}{n^3+n^2}$

$\boxed{\text{Comparison Test}}$

$a_n = \frac{n-1}{n^2(n+1)}$

$b_n = \frac{1}{n^2}$ which is convergent by p-series

$\frac{n-1}{n^2(n+1)} \le \frac{1}{n^2}$ for all n

so both are convergent

d) $\sum_{n=1}^{\infty}\frac{3^n\ n!}{n^4}$

$\boxed{\text{Ratio Test:}}$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{3^{n+1}(n+1)!}{(n+1)^4}\cdot\frac{n^4}{3^n n!}\right|$

$\lim_{n\to\infty}\left|\frac{3(n+1)}{1}\ \frac{n^4}{(n+1)^4}\right| = \infty(1)$

$\infty > 1$ so divergent

e) $\sum_{n=1}^{\infty} \frac{n(-3)^{n+1}}{7^n}$ $\frac{(-1)^n 3^n n}{7^n}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{3^{n+1}(n+1)}{7^{n+1}} \cdot \frac{7^n}{3^n n} \right| =$$

$$\lim_{n\to\infty} \left| \frac{3}{7} \cdot \frac{n+1}{n} \right| = 3/7 < 1$$

So absolutely convergent

f) $\sum_{n=1}^{\infty} \frac{n}{e^n}$ Ratio Test

$$\lim_{n\to\infty} \left| \frac{n+1}{e^{n+1}} \cdot \frac{e^n}{n} \right| =$$

$$\lim_{n\to\infty} \left| \frac{n+1}{n} \right| \cdot \lim_{n\to\infty} \left| \frac{1}{e} \right| =$$

$$1\left(\frac{1}{e}\right) = \frac{1}{e} < 1$$

So convergent

g) $\sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{(n+1)!}$ $= \frac{(-1)^{n-1} n^{n-1}}{(n+1)!}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{n^n}{(n+2)!} \cdot \frac{(n+1)!}{n^{n-1}} \right| =$$

$$\lim_{n\to\infty} \left| \frac{n}{n+2} \right| = 1$$

so inconclusive

Test for Divergence

Comparison Test

$a_n = \frac{10}{n^2}$ which is convergent by p-series

$b_n = \frac{n^{n-1}}{(n+1)!} \le \frac{10}{n^2}$ for all n

$\frac{1^0}{2!} \le \frac{10}{1}$

So absolutely convergent

h) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \ln n}{n}$

Integral Test

$$\lim_{t\to\infty} \int_1^t \ln\left(\frac{1}{n}\right) dn =$$

$$\lim_{t\to\infty} \left. \frac{(\ln n)^2}{2} \right|_1^t =$$

$$\lim_{t\to\infty} \left(\frac{(\ln t)^2}{2} - \frac{(\ln(1))^2}{2} \right) = \infty$$

so divergent