MATH 152 — QUIZ 7 — NAME Answers

Mrs. Bonny Tighe — 25 points — Wed. 11/2/05

12.4, 12.5, 12.6

1. Test the series for convergence or divergence. State the test you use and show all work. If the series is an Alternating Series, find if it is Absolutely or Conditionally convergent. Use each test at least one time. Ratio, Root, Integral + Comparison

a) $\sum_{n=2}^{\infty} \frac{(-1)^n(n-1)}{n^3+n} = \frac{n-1}{n^2(n+1)}$

$a_n = \frac{n-1}{n^2(n+1)}$

Comparison Test

Limit Comparison Test

$\frac{1}{n^2} = b_n$ which is convergent by p-series

$\lim_{n\to\infty} \left|\frac{a_n}{b_n}\right| = \lim_{n\to\infty} \left|\frac{\frac{n-1}{n^2(n+1)}}{\frac{1}{n^2}}\right|$

$\lim_{n\to\infty} \left|\frac{n-1}{n+1}\right| = 1 > 0$ so both

are convergent. absolutely

b) $\sum_{n=1}^{\infty} \frac{n(-2)^n}{5^{n-1}} = (-1)^n \frac{2^n n}{5^{n-1}}$

Ratio Test

$\lim_{n\to\infty} \left|\frac{2^{n+1}(n+1)}{5} \cdot \frac{5^{n-1}}{2^n n}\right| =$

$\lim_{n\to\infty} \left|\frac{n+1}{n} \frac{(2)}{5}\right| = 2/5 < 1$

So absolutely convergent

c) $\sum_{n=1}^{\infty} \frac{10^n}{n!}$

Ratio Test

$\lim_{n\to\infty} \left|\frac{10^{n+1}}{(n+1)!} \cdot \frac{n!}{10^n}\right|$

$\lim_{n\to\infty} \left|\frac{10}{n+1}\right| = 0 < 1$

So absolutely convergent

d) $\sum_{n=1}^{\infty} \frac{3^n\ n!}{n^4}$

Ratio Test

$\lim_{n\to\infty} \left|\frac{3^{n+1}(n+1)!}{(n+1)^4} \cdot \frac{n^4}{3^n(n!)}\right|$

$\lim_{n\to\infty} \left|3(n+1) \cdot \frac{n^4}{(n+1)^4}\right| = \infty$

$\infty > 0$

so Divergent.

or Test for divergence

$\frac{3(1)}{1}, \frac{9(2)}{16}, \frac{27(6)}{81} \rightarrow$

e) $\sum_{n=1}^{\infty} \frac{n}{e^n}$ $\boxed{\text{Ratio Test}}$

$$\lim_{n\to\infty} \left| \frac{n+1}{e^{n+1}} \cdot \frac{e^n}{n} \right|$$

$$\lim_{n\to\infty} \left| \frac{n+1}{n} \cdot \frac{1}{e} \right| = \frac{1}{e} < 1$$

So absolutely convergent

f) $\sum_{n=1}^{\infty} \left(\frac{-3n}{1+8n} \right)^n$ $\boxed{\text{Root Test}}$

$$\lim_{n\to\infty} \left(\frac{3n}{1+8n} \right) = \frac{3}{8} < 1$$

So convergent

absolutely convergent

g) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \ln n}{n}$

$\boxed{\text{Integral Test}}$

$$\lim_{n\to\infty} \int_1^n (\ln x)^1 \cdot \frac{1}{x}\, dx =$$

$$\lim_{n\to\infty} \frac{1}{2}(\ln x)^2 \Big|_1^n$$

$$\lim_{n\to\infty} \frac{1}{2}(\ln n)^2 - \frac{1}{2}(\ln 1)^2 =$$

$$\lim_{n\to\infty} \frac{(\ln n)^2}{2} = \infty$$

So divergent

AST - $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

$\boxed{\text{So conditionally convergent}}$

h) $\sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{(n+1)!}$ $\frac{(-1)^{n-1} n^{n-1}}{(n+1)!}$

$\boxed{\text{Comparison Test}}$

$a_n = \frac{10}{n^2}$ which is convergent by p-series, $p>1$

$\cancel{b_n = \frac{n^{n-1}}{(n+1)!} \leq \frac{10}{n^2}}$

$\cancel{\frac{1^{(0)}}{2!} \leq \frac{10}{1}}$

$\cancel{\frac{2^1}{3!}}$

$\frac{3^2}{4!}$

Test for Divergence

$$\lim_{n\to\infty} \left(\frac{n^{n-1}}{(n+1)!} \right) = \infty$$

So absolutely Divergent