MATH 152
Mrs. Bonny Tighe

**QUIZ 6**
8.8, 9.1, 9.2
25 points

NAME Answers

SECTION_______ ~~Tues. 3/29/05~~ Wed 10/19/05

1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

a) $\int_1^\infty \frac{x+1}{x^2+2x}\,dx$

$u = x^2+2x$
$du = 2x+2$
$2(x+1)$

$x(x+2)$

$\lim_{t\to\infty} \frac{1}{2}\ln x(x+2)\Big|_1^t =$

$\lim_{t\to\infty} \frac{1}{2}\ln t(t+2) - \frac{1}{2}\ln(1)(1+2) =$

$\lim_{t\to\infty} \ln\sqrt{\frac{t(t+2)}{3}} = \infty$ So Divergent

b) $\int_0^5 \frac{\ln x}{x}\,dx$

$\lim_{t\to 0} \int_t^5 (\ln x)'\left(\frac{1}{x}\,dx\right)$

$\lim_{t\to 0} \frac{1}{2}(\ln x)^2\Big|_t^5 =$

$\lim_{t\to 0} \frac{1}{2}(\ln(5))^2 - \left(\frac{1}{2}\ln t\right)^2 =$ DNE

So Divergent

c) $\int_{-\infty}^1 xe^{x^2}\,dx$

$\lim_{t\to -\infty} \frac{1}{2}e^{x^2}\Big|_t^1$

$\frac{1}{2}e^1 - \frac{1}{2}e^{t^2} = \frac{1}{2}e - \infty$

So Divergent

d) $\int_1^3 \frac{1}{y-1}\,dy$

$\lim_{t\to 1} \int_t^3 \frac{1}{y-1}\,dy$

$\lim_{t\to 1} \ln|y-1|\Big|_t^3$

$\lim_{t\to 1} \left(\ln(t-1) - \ln(2)\right)$ DNE

$\lim_{t\to 1} \left(\ln\left(\frac{t-1}{2}\right)\right)$ So Divergent

e) $\int_{-1}^1 \frac{e^m}{e^m-1}\,dm$

$e^m = u$
$e^m\,dm = du$
$dm = \frac{du}{u}$

$\int_{-1}^1 \left(\frac{u}{u-1}\right)\frac{du}{u}$

$\lim_{t\to 0} \int_{-1}^t \left(\frac{1}{u-1}\right)du = \lim_{t\to 0} \ln|u-1|\Big|_{-1}^t$

$\lim_{t\to 0} \left(\ln(e^0-1) - \ln(2)\right) =$ undefined

So divergent

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \qquad L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

2. Find the length of the curve on the given interval:

a) $y^2 = 4x$ , $0 \le y \le 2$

$2y = 4\,dx/dy$ = $dx/dy = \frac{1}{2}y$

$$L = \int_0^2 \sqrt{1+\tfrac{1}{4}y^2}\,dy$$

$$= \int_0^2 \frac{\sqrt{4+y^2}}{2}\,dy \qquad y = 2\tan\theta,\ dy = 2\sec^2\theta\,d\theta$$

$$\frac{1}{2}\int_0^2 \sqrt{4+4\tan^2\theta}\;2\sec^2\theta\,d\theta =$$

$$\frac{1}{\cancel{2}}\int_0^2 \cancel{2}\underbrace{\sec\theta}_{u}(\underbrace{2\sec^2\theta\,d\theta}_{dv}) = \tfrac{1}{2}\,2\int_0^2 \sec^3\theta\,d\theta.$$

$du = \sec\theta\tan\theta \qquad v = \tan\theta$

$$2\left[\sec\theta(\tan\theta) - \int_0^2 \tan\theta\sec\theta\tan\theta\,d\theta\right] = 2\sec\theta\tan\theta - \int_0^2 (\sec^3\theta - \sec\theta)d\theta$$

$\tan^2\theta\sec\theta$

b) $y = \ln(\sec x)$ , $0 \le x \le \pi/4$

$\frac{dy}{dx} = \frac{1}{\sec x}(\sec x\tan x)$

$$L = \int_0^{\pi/4} \sqrt{1+\tan^2 x}\,dx$$

$$\int_0^{\pi/4} \sec x\,dx =$$

$$\ln|\sec x + \tan x|\Big|_0^{\pi/4} =$$

$$\ln(\sec \pi/4 + \tan \pi/4) - \ln(\sec 0 + \tan 0)$$

$$\ln(\sqrt{2}+1) - \ln(1) =$$

$$L = \ln(\sqrt{2}+1)$$

3. a) Find the area of the surface obtained by rotating the curve about the y-axis.

$y = \sqrt[3]{x}$, $1 \le y \le 2$

$$\int_1^2 2\pi x\sqrt{1+(dx/dy)^2}\;dy$$

$$2\pi\int_1^2 y^3\sqrt{1+(3y^2)^2}\;dy$$

$$\frac{2\pi}{36}\int_1^2 \underbrace{\sqrt{1+9y^4}}_{u^{1/2}}\;\underbrace{36y^3\,dy}_{du}$$

$$\frac{\pi}{18}\,\frac{1}{3/2}u^{3/2}\Big|_1^2 = \frac{\pi}{18}\cdot\frac{2}{3}(1+9y^4)^{3/2}\Big|_1^2$$

$$\frac{\pi}{27}(1+9(16))^{3/2} - \frac{\pi}{27}(1+9)^{3/2}$$

$$\pi/27\,(145\sqrt{145} - 10\sqrt{10})$$

b) Find the surface area obtained by rotation of the curve about the x-axis

$y = \sqrt{x}$, $4 \le x \le 9$

$$\int_4^9 2\pi y\sqrt{1+(dy/dx)^2}\;dx$$

$$2\pi\int_4^9 \sqrt{x}\sqrt{1+\left(\tfrac{1}{2\sqrt{x}}\right)^2}\;dx$$

$$2\pi\int_4^9 \sqrt{x}\sqrt{1+1/4x} = \sqrt{x+\tfrac{1}{4}}\;dx$$

$$\int u^{1/2}\,du$$

$$2\pi\,\frac{1}{3/2}u^{3/2}\Big|_4^9 = \frac{4}{3}\pi(x+\tfrac{1}{4})^{3/2}\Big|_4^9$$

$$\frac{4\pi}{3}\left[(9\tfrac{1}{4})^{3/2} - (4\tfrac{1}{4})^{3/2}\right] =$$

$$\frac{4\pi}{3}\left[\frac{37}{4}\sqrt{\frac{37}{4}} - \frac{17}{4}\sqrt{\frac{17}{4}}\right]$$