$\int u\,dv = uv - \int v\,du$

MATH 152
Mrs. Bonny Tighe

QUIZ 5A
25 points
8.5, 8.6, 8.7

NAME Answers

SECTION ______ Wed 10/12/05

1. Evaluate the integrals.

a) $\int x\,(3^x)\,dx$

$u = x$, $dv = 3^x\,dx$

$du = dx \quad v = \dfrac{3^x}{\ln 3}$

$x\left(\dfrac{3^x}{\ln 3}\right) - \int \dfrac{3^x}{\ln 3}\,dx$

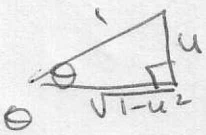
$\dfrac{x3^x}{\ln 3} - \dfrac{1}{\ln 3}\int 3^x\,dx$

$\dfrac{x3^x}{\ln 3} - \dfrac{1}{\ln 3}\cdot\dfrac{3^x}{\ln 3} + C$

$\dfrac{x3^x}{\ln 3} - \dfrac{3^x}{(\ln 3)^2} + C$

b) $\int x\sqrt{1-x^4}\,dx \qquad x^2 = u,\ 2x\,dx = du$

$\frac{1}{2}\int 2x\sqrt{1-(x^2)^2}\,dx$



$\frac{1}{2}\int \sqrt{1-u^2}\,du \rightarrow u = \sin\theta$
$du = \cos\theta\,d\theta$

$\frac{1}{2}\int \sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta = \frac{1}{2}\int \cos^2\theta\,d\theta$

$\frac{1}{2}\int \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\,d\theta\right) =$

$\int \frac{1}{4} + \frac{1}{4}\cos 2\theta\,d\theta = \frac{1}{4}\theta + \frac{1}{8}\sin 2\theta + C$

$\frac{1}{4}\theta + \frac{1}{4}\sin\theta\cos\theta = \frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}(x^2)\left(\frac{\sqrt{1-x^4}}{1}\right) + C$

c) $\int_0^{\pi/4} \cos^3\vartheta\,d\vartheta$

$\int_0^{\pi/4} \cos^2\theta\ \cos\theta\,d\theta$
$(1-\sin^2\theta)$

$\int_0^{\pi/4}\cos\theta\,d\theta - \int_0^{\pi/4}\sin^2\theta\cos\theta\,d\theta$

$\sin\theta - \frac{1}{3}\sin^3\theta\,\Big|_0^{\pi/4}$

$\sin\pi/4 - \frac{1}{3}(\sin\pi/4)^3 - (0-0)$

$\sqrt{2}/2 - \frac{1}{3}(\sqrt{2}/2)^3$

$\sqrt{2}/2 - \frac{2}{4}\cdot\frac{1}{3}\sqrt{2}/2$

$\sqrt{2}/2 - \frac{1}{6}\sqrt{2}/2$

$\dfrac{5\sqrt{2}}{6}$

d) $\int \dfrac{x^2}{(x+2)^2}\,dx = \int 1\,dx - 4\int \dfrac{x-1}{(x+2)^2}\,dx$

$\dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} = \dfrac{x-1}{(x+2)^2}$

$A(x+2) + B = x - 1 \qquad A = 1$

$Ax + 2A + B = x - 1 \qquad B = -3$

$\int 1\,dx - 4\cdot\int \dfrac{1}{x+2}\,dx - 4(-3)\int \dfrac{1}{(x+2)^2}\,dx$

$x - 4\ln|x+2| + 12\cdot\dfrac{1}{-1}(x+2)^{-1} + C$

$x - 4\ln|x+2| - \dfrac{12}{x+2} + C$

$$\begin{array}{r} 1 \\ x^2+4x+4\ \overline{)\ x^2} \\ -(x^2+4x+4) \\ \hline -4(x+1) \end{array}$$

2. Use (a) the Midpoint Rule (b) the Trapezoidal Rule and (c) Simpson's Rule to approximate the given integral with the specified value of n. Estimate the errors in the approximations M_4 and T_4 given the following: Suppose $|f''(x)| \le K$ for $a \le x \le b$,

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \quad and \quad |E_M| \le \frac{K(b-a)^3}{24n^2}. \qquad \int_0^{1/2} \sin(x^2)\,dx, \quad n = 4$$

$$\Delta x = \frac{1/2 - 0}{4} = \frac{1}{8}$$

$$a)\ M_4 = \frac{1}{8}\left[f(1/16) + f(3/16) + f(5/16) + f(7/16)\right]$$

$$= \frac{1}{8}\left[\sin\left(\tfrac{1}{16}\right)^2 + \sin(3/16)^2 + \sin(5/16)^2 + \sin\left(\tfrac{7}{16}\right)^2\right]$$

$$b)\ T_4 = \frac{1}{16}\left[f(0) + 2f(1/8) + 2f(1/4) + 2f(3/8) + f(1/2)\right]$$

$$= \frac{1}{16}\left[\sin 0 + 2\sin(1/8)^2 + 2\sin(1/4)^2 + 2\sin(3/8)^2 + \sin(1/2)\right]$$

$$c)\ S_4 = \frac{1}{24}\left[f(0) + 4f(1/8) + 2f(1/4) + 4f(3/8) + f(1/2)\right]$$

$$= \frac{1}{24}\left[\sin 0 + 4\sin(1/8)^2 + 2\sin(1/4)^2 + 4\sin(3/8)^2 + \sin(1/2)^2\right]$$

$$f'(x) = \cos(x^2)(2x)$$

$$f''(x) = \cos x^2(2) + 2x(-\sin(x^2))(2x)$$

$$= 2\cos x^2 - 4x^2\sin(x^2)$$

$$f''(0) = 2\cos(0)^2 - 4(0)(0) = 2 \text{ Max}$$

$$f''(1/2) = 2\cos(1/4) - 4\left(\tfrac{1}{4}\right)\sin\left(\tfrac{1}{4}\right)$$

$$2\cos 1/4 - \sin 1/4$$

$$|E_T| \le \frac{2(1/2 - 0)^3}{12(4)^2} = \frac{2\left(\frac{1}{8}\right)}{12 \cdot 16} = \frac{1}{6} \cdot \frac{1}{8 \cdot 16)}$$

$$|E_T| \le \frac{1}{768}$$

$$|E_M| \le \frac{1}{768(2)} = \frac{1}{1536}$$