MATH 152 | **QUIZ 4A** | NAME Answers

Mrs. Bonny Tighe | 25 points

8.3, 8.4 | SECTION ________ Wed 10/5/05

1. Evaluate the integrals.

a) $\int \frac{x^2}{\sqrt{16-x^2}}dx = \int \frac{16\sin^2\theta}{\sqrt{16-16\sin^2\theta}}(4\cos\theta d\theta) =$

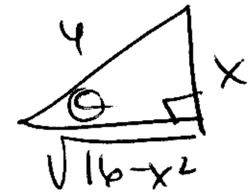
$x = 4\sin\theta$

$dx = 4\cos\theta d\theta$

$\int \frac{16\sin^2\theta \, 4\cancel{\cos\theta} d\theta}{4\cancel{\cos\theta}} = 16\int \sin^2\theta d\theta = 16\int\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta =$

$\int 8 - 8\cos 2\theta d\theta = 8\theta - 4\sin 2\theta + C =$

$8\theta - 4(2\sin\theta\cos\theta) + C$



$\sin^{-1}\left(\frac{x}{4}\right) - 8\left(\frac{x}{4}\right)\left(\frac{\sqrt{16-x^2}}{x}\right) + C$

$\sin^{-1}\left(\frac{x}{4}\right) - 2x\left(\frac{\sqrt{16-x^2}}{x}\right) + C = \sin^{-1}\left(\frac{x}{4}\right) - 2\sqrt{16-x^2} + C$

b) $\int \frac{x\,dx}{\sqrt{x^2-6x+10}} = \int \frac{u+3\,du}{\sqrt{u^2+1}} = \frac{1}{2}\int \frac{2u}{\sqrt{u^2+1}} + \frac{3}{\sqrt{u^2+1}}du$

$x^2 - 6x + 10$

$(x-3)^2 + 1$

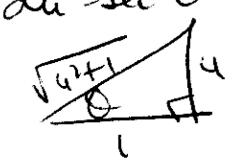
$u = x-3$

$du = dx$

$u+3 = x$

$u = \tan\theta$

$du = \sec^2\theta d\theta$



$\frac{1}{2}\cdot\frac{1}{1/2}(u^2+1)^{1/2} + \int \frac{3\sec^2\theta d\theta}{\sqrt{\tan^2\theta+1}}$

$\sqrt{u^2+1} + 3\int \sec\theta d\theta = \sqrt{u^2+1} + 3\ln|\sec\theta + \tan\theta| + C$

$\sqrt{u^2+1} + 3\ln|\sqrt{u^2+1} + u| + C$

$\sqrt{(x-3)^2+1} + 3\ln\left|\sqrt{(x-3)^2+1} + (x-3)\right| + C$

2. Evaluate the integral.

a) $\int \frac{x^3+2x^2+1}{x^3-3x^2}dx$

first

$$\begin{array}{r} 1 \\ x^3-3x^2 \overline{\big)\, x^3+2x^2+1} \\ -(x^3-3x^2) \\ \hline 5x^2+1 \end{array}$$

$$\int 1 + \frac{5x^2+1}{x^2(x-3)}dx = \int 1dx + -\frac{1}{9}\int \frac{1}{x}dx - \frac{1}{3}\int \frac{1}{x^2}dx + \frac{46}{9}\int \frac{1}{x-3}dx$$

$$x - \frac{1}{9}\ln|x| - \frac{1}{3}\left(\frac{1}{-1}x^{-1}\right) + \frac{46}{9}\ln|x-3| + C$$

$$x - \frac{1}{9}\ln|x| + \frac{1}{3x} + \frac{46}{9}\ln|x-3| + C$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} = \frac{5x^2+1}{x^2(x-3)}$$

$$Ax(x-3) + B(x-3) + Cx^2 = 5x^2+1$$

$x=3$: $9C = 45+1$ $\quad C = 46/9$

$x=0$: $-3B = 1$ $\quad B = -1/3$

$Ax^2 + Cx^2 = 5x^2$

$A + 46/9 = 5 = 45/9$

$A = -1/9$

b) $\int \frac{\cos\theta}{\sin^2\theta + 4\sin\theta - 5}d\theta$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$\int \frac{du}{u^2+4u-5} = \int \frac{-\frac{1}{6}}{u+5}du + \frac{+\frac{1}{6}}{u-1}du =$$

$(u+5)(u-1)$

$$-\frac{1}{6}\int \frac{1}{u+5}du + \frac{1}{6}\int \frac{1}{u-1}du =$$

$$\frac{A}{u+5} + \frac{B}{u-1} = \frac{1}{(u+5)(u-1)}$$

$$-\frac{1}{6}\ln|u+5| + \frac{1}{6}\ln|u-1| + C$$

$$A(u-1) + B(u+5) = 1$$

$$-\frac{1}{6}\ln|\sin\theta+5| + \frac{1}{6}\ln|\sin\theta-1| + C$$

$u=1$: $6B = 1$, $B = 1/6$

$u=-5$: $-6A = 1$ $\quad A = -1/6$