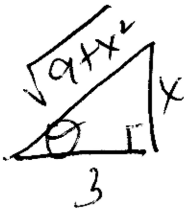
MATH 152 **QUIZ 4** NAME Answers

Mrs. Bonny Tighe 25 points

8.3, 8.4 Section ______ Wed 10/5/05



1. Evaluate the integrals.

a) $\int \frac{\sqrt{9+x^2}}{x}dx$ $= \int \frac{\sqrt{9+9\tan^2\theta}}{3\tan\theta} 3\sec^2\theta d\theta = \int \frac{3\sec\theta\, 3\sec^2\theta}{3\tan\theta} d\theta$

$x = 3\tan\theta$

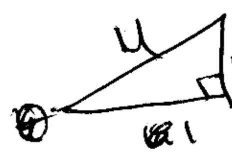
$dx = 3\sec^2\theta\, d\theta$ $\quad 3\int \frac{\sec^3\theta}{\tan\theta} d\theta = 3\int \sec^3\theta \cot\theta\, d\theta$

$= 3\int \sec\theta(1+\tan^2\theta)\cot\theta\, d\theta = 3\int \sec\theta\cot\theta + \sec\theta\tan^2\theta\cot\theta\, d\theta$

$= 3\int \frac{1}{\cos\theta}\cdot\frac{\cos}{\sin\theta} + \sec\theta\tan\theta\, d\theta = 3\int \csc\theta + \sec\theta\tan\theta\, d\theta$

$3\ln|\csc\theta - \cot\theta| + \sec\theta + C$

$3\ln\left|\frac{\sqrt{9+x^2}}{x} - \frac{3}{x}\right| + \frac{\sqrt{9+x^2}}{3} + C$



$u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

b) $\int \frac{x\, dx}{\sqrt{x^2+6x+8}} =$ ______________

$(x+3)^2 - 1$

$u = x+3,\ x = u-3$

$du = dx$

$\int \frac{u-3\, du}{\sqrt{u^2-1}} = \frac{1}{2}\int \frac{2u}{\sqrt{u^2-1}} - \int \frac{3}{\sqrt{u^2-1}} du$

$\int \frac{\sec\theta\tan\theta\, d\theta}{\sqrt{\sec^2\theta - 1}}$

$\frac{1}{2}\int w^{-1/2} dw \quad -3\int \sec\theta\, d\theta$

$\frac{1}{2}\cdot\frac{1}{1/2}w^{1/2}$

$\sqrt{u^2-1} \quad -3\ln|\sec\theta + \tan\theta| + C$

$\sqrt{u^2-1} - 3\ln\left|u + \sqrt{u^2-1}\right| + C$

$\sqrt{(x+3)^2-1} - 3\ln\left|(u+3) + \sqrt{(u+3)^2-1}\right| + C$

$$x^3 - x \overline{\big) \begin{array}{r} 1 \\ x^3 + 2 \\ -(x^3 \quad -x) \\ \hline x + 2 \end{array}}$$

2. Evaluate the integral.

a) $\int \frac{x^3+2}{x^3-x}dx =$

$$\int 1 + \frac{x+2}{x(x+1)(x-1)} dx = \int dx + \cancel{2}\int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{x+1}dx + \frac{3}{2}\int \frac{1}{x-1}dx$$

$$x - 2\ln|x| + \frac{1}{2}\ln|x+1| + \frac{3}{2}\ln(x-1) + C$$

$$\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} = \frac{x+2}{x(x+1)(x-1)}$$

$A(x+1)(x-1) + B(x)(x-1) + C(x)(x+1) = x+2$

$x = 1 \qquad C(1)(2) = 3 \qquad C = 3/2$

$x = -1 \qquad B(-1)(-2) = 1 \qquad B = 1/2$

$Ax^2 - A + Bx^2 - Bx + Cx^2 + Cx = x + 2, \; A = -2$

$u = e^x \qquad du = e^x dx$ so $dx = \frac{du}{u}$

b) $\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx$

$$\int \frac{u^2}{u^2+3u+2}\left(\frac{du}{u}\right) = \int \frac{u\,du}{(u+2)(u+1)} = \int \frac{2}{u+2}du - \int \frac{1}{u+1}du$$

$$2\ln|u+2| - \ln|u+1| + C$$

$$2\ln|e^x+2| - \ln|e^x+1| + C$$

$$\frac{A}{u+2} + \frac{B}{u+1} = u$$

$A(u+1) + B(u+2) = u$

$(u = -1) \qquad B = -1$

$(u = -2) \qquad -A = -2, \; A = 2$