MATH 152
Mrs. Bonny Tighe

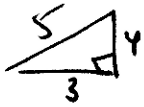
QUIZ 3A

7.5-7.7
25 points

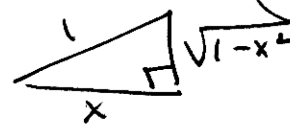
NAME Answers

SECTION________ Wed 9/21/05

1. Simplify: a) $\sec(\tan^{-1} 4/3) =$ $5/3$



b) $\csc(\cos^{-1} x) =$ $1/\sqrt{1-x^2}$



2. Differentiate the following.

a) $y = \tanh^{-1}(\sin(\cos x))$

$$\frac{dy}{dx} = \frac{1}{1-(\sin(\cos x))^2}(\cos(\cos x))(-\sin x)$$

$$= \frac{-\sin x \cos(\cos x)}{1-\sin^2(\cos x)}$$

b) $f(x) = \operatorname{arccot}^3(x)$

$$f'(x) = 3\operatorname{arccot}^2(x)\cdot\left(-\frac{1}{1+x^2}\right)$$

$$f'(x) = \frac{-3\operatorname{arccot}^2 x}{1+x^2}$$

~~c) $f(x) = \cosh^{-1}\sqrt{x}$~~

$$f'(x) = \frac{1}{\sqrt{x-1}}\cdot\frac{1}{2\sqrt{x}}$$

~~d) $f(x) = (\sinh^{-1} x)(\tan^{-1} x)$~~

$$f'(x) = \sinh^{-1}x\left(\frac{1}{1+x^2}\right) + \tan^{-1}x\left(\frac{1}{\sqrt{1+x^2}}\right)$$

3. Evaluate: a) $\int \frac{\tan^{-1} x}{1+x^2}dx =$ ________

$$\int \tan^{-1}x \cdot \frac{1}{1+x^2}dx = \int u^1 du$$

$$\frac{1}{2}u^2 + C = \frac{1}{2}(\tan^{-1}x)^2 + C$$

b) $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}dx$

$u = e^{2x}$
$du = e^{2x}(2dx)$
$\frac{du}{2u} = dx$

$$\int \frac{u}{\sqrt{1-u^2}}\left(\frac{du}{2u}\right)$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}du = \frac{1}{2}\sin^{-1}(e^{2x}) + C$$

c) $\int \frac{1}{x\sqrt{x^2-1}}dx =$ $-\csc^{-1}x + C$

d) $\int_0^1 \frac{1}{\sqrt{9x^2+1}}dx =$ ________

$u = 3x$
$du = 3$

$$\frac{1}{3}\int_0^1 \frac{1}{\sqrt{(3x)^2+1}}3dx$$

$$\frac{1}{3}\sinh^{-1}(3x)\Big|_0^1 =$$

$$\frac{1}{3}\sinh^{-1}(3) - \frac{1}{3}\sinh^{-1}(0)$$

4. Find the limit. Use L'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If L'Hospital's Rule doesn't apply, explain why.

0^0 a) $\lim_{x\to 0^+} (\tan 2x)^x = e^0 = 1$

$\ln y = \lim_{x\to 0^+} x\ln(\tan 2x) = 0\cdot\infty$

$\lim_{x\to 0^+}\left(\frac{\ln(\tan 2x)}{\frac{1}{x}}\right) = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to 0^+} \frac{\frac{1}{\tan 2x}\sec^2 2x\,(2)}{-1/x^2} = \lim_{x\to 0^+}\left(\frac{-2x^2}{\sin 2x\cos 2x}\right) = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{-4x}{(\sin 2x)(-\sin 2x)(2) + \cos 2x(\cos 2x)(2)} = \frac{0}{2} = 0,\ e^0 = 1$

b) $\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^x = e^{-2}$

$\ln y = \lim_{x\to\infty} x\ln\left(\frac{x-1}{x+1}\right) =$

$\ln y = \lim_{x\to\infty}\left(\frac{\ln\left(\frac{x-1}{x+1}\right)}{\frac{1}{x}}\right) = \frac{0}{0} \overset{L'H}{=}$

$\lim_{x\to\infty}\left(\frac{\frac{1}{x-1} - \frac{1}{x+1}}{-1/x^2}\right) =$

$\lim_{x\to\infty}\left(\frac{-x^2(x+1) + x^2(x-1)}{x^2-1}\right) =$

$\lim_{x\to\infty}\left(\frac{-x^2 - x^2}{x^2-1}\right) = -2 = \ln y$

c) $\lim_{t\to 0}\frac{1-e^{3t}}{t} = -3$

$\lim_{t\to 0}\frac{1-e^{3t}}{t} = \frac{0}{0} \overset{L'H}{=}$

$\lim_{t\to 0}\frac{-3e^{3t}}{1} = -3e^0 = -3$

d) $\lim_{x\to 0^+} x\ln x = 0$

$0\cdot\infty =$

$\lim_{x\to 0^+}\left(\frac{\ln x}{\frac{1}{x}}\right) = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to\infty}\left(\frac{\frac{1}{x}}{-1/x^2}\right) = \lim_{x\to\infty}(-x) = 0$

e) $\lim_{x\to 0^+}\sin x\cot 2x =$ _______

$0\cdot\infty$

$\lim_{x\to 0^+}\frac{\sin x}{\tan 2x} = \frac{0}{0} \overset{L'H}{=}$

$\lim_{x\to 0^+}\frac{\cos x}{\sec^2 2x} = 1$

5. Find the numerical value of each expression:

a) $\tan(\sin^{-1}\sqrt{2}/2) = 1$ b) $\cosh(0) = 1$ c) $\sin^{-1}(1) = \pi/2$

$\frac{e^x + e^{-x}}{2}$

$\frac{e^0 + e^{-0}}{2}\ \frac{2}{2} = 1$