MATH 152 | **QUIZ 3** | NAME Answers
Mrs. Bonny Tighe | 7.5-7.7 |
25 points | SECTION________Wed 9/21/05

1. Differentiate the following.

a) $y = \cosh^{-1}(\cos^{-1}\sqrt{x})$

$$\frac{dy}{dx} = \frac{1}{\sqrt{(\cos^{-1}\sqrt{x})^2 - 1}}\left(-\frac{1}{\sqrt{1-x}}\right)\left(\frac{1}{2\sqrt{x}}\right)$$

b) $f(x) = \arctan(\csc x)$

$$f'(x) = \frac{1}{1+\csc^2 x}(-\csc x \cot x) \text{ or }$$

$$\frac{-\csc x \cot x}{1+\csc^2 x}$$

c) $f(x) = \sin^{-1}(x^3)$

$$f'(x) = \frac{1}{\sqrt{1-x^6}}(3x^2)$$

d) $f(x) = (\cos^{-1}x)(\tanh^{-1}x)$

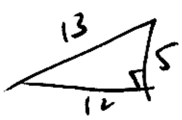
$$f'(x) = (\cos^{-1}x)\left(\frac{1}{1-x^2}\right) + \tanh^{-1}x\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

2. Find the numerical value of each expression:

a) $\cos(\sin^{-1}\sqrt{3}/2) =$ $\frac{1}{2}$ b) $\sinh(0) =$ 0 c) $\cos^{-1}(1) =$ 0

$$\frac{e^x - e^{-x}}{2} = \frac{e^0 - e^0}{2}$$

3. Simplify: a) $\sin(\tan^{-1}5/12) =$ $5/13$ b) $\sin(\tan^{-1}x) =$ $\frac{x}{\sqrt{1+x^2}}$



4. Evaluate: a) $\int \frac{\tan^{-1}x}{1+x^2}dx =$ ______

$$\int (\tan^{-1}x)^1 \cdot \frac{1}{1+x^2}\,dx$$

$$\int u^1\,du = \frac{1}{2}u^2 + C$$

$$\frac{1}{2}(\tan^{-1}x)^2 + C$$

b) $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}dx$

$u = e^{2x}$

$du = 2e^{2x}dx$

$\frac{du}{2e^{2x}} = dx$

$\frac{du}{2u} = dx$

$$\int \frac{\cancel{u}}{\sqrt{1-u^2}} \cdot \frac{du}{2\cancel{u}}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\sin^{-1}u + C$$

$$\frac{1}{2}\sin^{-1}(e^{2x}) + C$$

$$\int \frac{1}{(3x)\sqrt{(3x)^2-1}}(3dx) \qquad \frac{1}{4}\int_0^1 \frac{1}{\sqrt{(4x)^2+1}}4dx =$$

c) $\int \frac{1}{x\sqrt{9x^2-1}}dx =$ ________

$\sec^{-1}(3x)+C$

d) $\int_0^1 \frac{1}{\sqrt{16x^2+1}}dx =$ ________

$\frac{1}{4}\sinh^{-1}(4x)\Big|_0^1 = \frac{1}{4}\sinh^{-1}(4) - \frac{1}{4}\sinh^{-1}(0)$

5. Find the limit. Use L'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If L'Hospital's Rule doesn't apply, explain why.

a) $\lim_{x\to 0^+}(\tan 3x)^x =$ $e^0 = 1$

0^0

$\ln y = \lim_{x\to 0^+} x(\ln \tan 3x)$

$\ln y = \lim_{x\to 0^+} \frac{\ln(\tan 3x)}{\frac{1}{x}} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to 0^+} \frac{\frac{1}{\tan 3x}(\sec^2 3x)(3)}{-1/x^2} =$

$\lim_{x\to 0^+} \frac{-x^2(3\sec^2 3x)}{\tan 3x}$ $\lim_{x\to 0^+}\frac{-3x^2}{\cos 3x \sin 3x} = \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0^+}\frac{-6x}{(\cos 3x)(\cos 3x)(3)+(\sin 3x)(-\sin 3x)(3)} = \frac{0}{3} = 0$

$e^0 = 1$

b) $\lim_{x\to\infty}\frac{x^3}{e^x} =$ ∞

$\frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to\infty}\frac{3x^2}{e^x} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to\infty}\frac{6x}{e^x} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to\infty}\frac{6}{e^x} = \infty$

c) $\lim_{t\to 0}\frac{1-e^{2t}}{t} =$ -2

$\frac{0}{0} \overset{L'H}{=}$

$\lim_{t\to 0}\frac{-2e^{2t}}{1} = -2e^0 = -2$

d) $\lim_{x\to 0^+} x\ln x =$ 0

$0\cdot\infty = \lim_{x\to 0^+}\left(\frac{\ln x}{\frac{1}{x}}\right) = \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{\frac{-1}{x^2}}\right) = \lim_{x\to 0^+}(-x) = 0$

e) $\lim_{x\to\infty}\left(\frac{x+1}{x+2}\right)^x =$ e^{-1} or $\frac{1}{e}$

0^∞ $y = \lim_{x\to\infty}\left(\frac{x+1}{x+2}\right)^x$

$\ln y = \lim_{x\to\infty} x\ln\left(\frac{x+1}{x+2}\right) = \infty\cdot 0$

$\ln y \overset{L'H}{=} \lim_{x\to\infty}\left(\frac{\ln\left(\frac{x+1}{x+2}\right)}{\frac{1}{x}}\right) \overset{L'H}{=}$

$\lim_{x\to\infty}\left(\frac{\frac{1}{x+1}-\frac{1}{x+2}}{-\frac{1}{x^2}}\right) =$

$\lim_{x\to\infty}\left(\frac{-x^2(x+2)-(-x^2)(x+1)}{(x+1)(x+2)}\right) =$

$\ln y = \lim_{x\to\infty}\left(\frac{-x^2}{x^2+3x+2}\right) = -1$ so e^{-1}