

1. Differentiate.

a) $f(x) = \ln(\sin^4 x)$

$$f'(x) = \frac{1}{\sin^4 x} \cdot 4 \sin^3 x \cdot (\cos x)$$

$$= 4 \frac{\cos x}{\sin x} = 4 \cot x$$

b) $y = \ln\left(\frac{\sqrt{x+1}(\sec 3x)}{(2x-1)^4}\right)$

$$y = \frac{1}{2} \ln(x+1) + \ln(\sec 3x) - 4 \ln(2x-1)$$

$$\frac{dy}{dx} = \frac{1}{2(x+1)} + \frac{1}{\sec 3x} (\sec 3x \tan 3x)(3) - \frac{4(2)}{2x-1}$$

$$\left(\frac{dy}{dx} = \frac{1}{2(x+1)} + 3 \tan 3x - \frac{8}{2x-1} \right)$$

2. Find the exact value: a) $3^{3 \log_3 2} =$ _____ b) $\log_2 22 - \log_2 11 =$ _____

$$3^{\log_3 8} = 8$$

$$\log_2 \frac{22}{11} = \log_2 (2) = 1$$

3. Solve for x.

a) $\log_5 (2-2x) = 2$

$$5^2 = 2-2x$$

$$2x = 2-25$$

$$x = -23/2$$

b) $4e^{2x-1} = 2$

$$e^{2x-1} = 1/2$$

$$\ln 1/2 = 2x-1$$

$$\frac{\ln 1/2 + 1}{2} = x$$

c) $2e^{2x} + e^x - 3 = 0$

$$(2e^x + 3)(e^x - 1) = 0$$

$$e^x = -3/2 \quad e^x = 1$$

$$x = 0$$

4. Find an equation of the tangent line to the curve $y = \log_2 x^3$ at the point (2,3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 = (x - 2)$$

$$y - 3 = \frac{3}{2 \ln 2} (x - 2) \quad \text{or}$$

$$y = \frac{3}{2 \ln 2} x - \frac{6}{2 \ln 2} + 3$$

$$m = \frac{dy}{dx}$$

$$y = 3 \log_2 x$$

$$\frac{dy}{dx} = \frac{3}{x \ln 2} \quad \text{at } x = 2$$

$$= \frac{3}{2 \ln 2}$$

5. Evaluate:

a) $\int_0^2 \frac{x^2}{3+x^3} dx =$ _____

$$\frac{1}{3} \int_0^2 \frac{1}{u} du \quad u = 3+x^3$$

$$du = 3x^2 dx$$

$$\frac{1}{3} \ln|3+x^3| \Big|_0^2 =$$

$$\frac{1}{3} \ln(11) - \frac{1}{3} \ln(3)$$

b) $\int \frac{(\ln x)}{x} dx =$ _____

$$\int u' du$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\frac{1}{2} (\ln x)^2 + C$$

c) $\int \frac{2^{3x}}{3-2^{3x}} dx =$ _____

$$\int \frac{1}{u} du$$

$$u = 3-2^{3x}$$

$$du = -3 \cdot 2^{3x} \ln 2 dx$$

$$-\frac{1}{3 \ln 2} \ln|3-2^{3x}| + C$$

d) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$ _____

$$2 \int e^u du$$

$$u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$2e^{\sqrt{x}} + C$$

6. Use logarithmic differentiation to find the derivative of a) $y = x^{\cos 3x}$

$$\ln y = \cos 3x \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = \cos 3x \cdot \frac{1}{x} + \ln x (-\sin 3x)(3) \quad \frac{dy}{dx} = \left(\frac{\cos 3x}{x} - 3 \sin 3x \ln x \right) x^{\cos 3x}$$

b) $y = \sqrt{\frac{2x-1}{\sin x}}$

$$\ln y = \frac{1}{2} \ln(2x-1) - \frac{1}{2} \ln(\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{2x-1} (2) - \frac{1}{2} \cdot \frac{1}{\sin x} (\cos x)$$

$$\frac{dy}{dx} = \left(\frac{1}{2x-1} - \frac{1}{2} \cot x \right) \left(\sqrt{\frac{2x-1}{\sin x}} \right)$$