MATH 152 **QUIZ 1A** NAME Answers
Mrs. Bonny Tighe 7.1-7.2 Wednesday 9/7/05
25 points

1. Find a formula for the inverse of the function $f(x) = \frac{3x-1}{x+3}$

$x = \frac{3y-1}{y+3}$

$xy + 3x = 3y - 1$

$3x + 1 = 3y - xy$

$y(3-x) = 3x+1$

$y = \frac{3x+1}{3-x}$ or $\frac{-3x-1}{x-3}$

2. Find dy/dx if $ye^x = y^3 + e^{xy}$

$y \cdot e^x + e^x \, dy/dx = 3y^2 \, dy/dx + e^{xy}\left(x \, dy/dx + y(1)\right)$

$e^x \, dy/dx - 3y^2 \, dy/dx - xe^{xy} \, dy/dx = ye^{xy} - ye^x$

$dy/dx = \frac{ye^{xy} - ye^x}{e^x - 3y^2 - xe^{xy}}$

3. Find the derivative: a) $y = (3e^{\sqrt{x}})\left(\frac{1}{x^3}\right)$

$dy/dx = (3e^{\sqrt{x}})(-3x^{-4}) + \frac{1}{x^3} \cdot 3e^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)$

$= -\frac{9e^{\sqrt{x}}}{x^4} + \frac{3e^{\sqrt{x}}}{2x^3\sqrt{x}}$

b) $g(x) = \frac{e^{\csc x}}{2-\sqrt{x}}$

$g'(x) = \frac{(2-\sqrt{x})(e^{\csc x})(-\csc x \cot x) - (e^{\csc x})\left(-\frac{1}{2\sqrt{x}}\right)}{(2-\sqrt{x})^2}$

$g'(x) = \frac{-2\sqrt{x}(2-\sqrt{x})e^{\csc x}(\csc x \cot x) + e^{\csc x}}{2\sqrt{x}(2-\sqrt{x})^2}$

$f(?) = 2$ so $f^{-1}(2) = 2$

$f(1) = 2$ so $f^{-1}(2) = 1$

$g(x) = f^{-1}(x)$

4. Find $(f^{-1})'(a)$ for $f(x) = \dfrac{x+1}{2-x}$ at $a = 2$.

$\dfrac{x+1}{2-x} = 2$

$x+1 = 4-2x$

$3x = 3$

$x = 1$

$$\boxed{g'(a) = \frac{1}{f'(g(a))}}$$

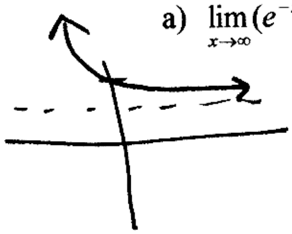
$$f'(x) = \frac{(2-x)(1)-(x+1)(-1)}{(2-x)^2} = \frac{2-x+x+1}{(2-x)^2}$$

$$f'(x) = \frac{3}{(2-x)^2} = \frac{3}{(2-1)^2} = 3$$

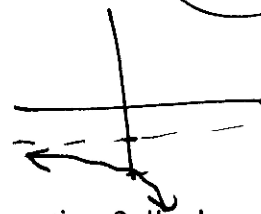
$$= \frac{1}{3} = 1/3$$

5. Find the limit.

a) $\lim_{x\to\infty}(e^{-x}+1) =$ 1 b) $\lim_{x\to\infty}\dfrac{e^x-e^{-x}}{e^x+e^{-x}} =$ 1 c) $\lim_{x\to-\infty}(-e^{2x}-1) =$ -1



$$\lim_{x\to\infty}\frac{e^{2x}-1}{e^{2x}+1} = \frac{1-\frac{1}{e^{2x}}}{1+\frac{1}{e^{2x}}}$$



6. Show that the function $y = e^x + e^{-x/2}$ satisfies the differential equation $2y''-y'-y = 0$

$$y' = e^x + e^{-x/2}(-1/2) = e^x - \frac{1}{2}e^{-x/2}$$

$$y'' = e^x - \frac{1}{2}\cdot e^{-x/2}(-1/2) = e^x + \frac{1}{4}e^{-x/2}$$

$$2\left(e^x+\frac{1}{4}e^{-x/2}\right) - \left(e^x-\frac{1}{2}e^{-x/2}\right) - \left(e^x+e^{-x/2}\right)$$

$$2e^x + \frac{1}{2}e^{-x/2} - e^x + \frac{1}{2}e^{-x/2} - e^x - e^{-x/2} = 0$$

yes

7. Evaluate the integral:

a) $\int e^{-x}\sqrt{e^{-x}+3}\,dx$ b) $\int e^{3x}\sec(e^{3x})\tan(e^{3x})dx$ c) $\int_1^4 \dfrac{e^{\sqrt{x}}}{\sqrt{x}}dx$

$u = e^{\sqrt{x}}$

$du = \dfrac{1}{2\sqrt{x}}dx$

a) $u = e^{-x}+3$, $du = -e^{-x}dx$

$$-\int u^{1/2}\,du$$

$$-\frac{1}{3/2}u^{3/2}+C$$

$$\boxed{-\frac{2}{3}(e^{-x}+3)^{3/2}+C}$$

b) $u = e^{3x}$, $du = e^{3x}(3)\,dx$

$$\frac{1}{3}\int \sec u \tan u\,du$$

$$\frac{1}{3}\sec(e^{3x})+C$$

c)

$$2\int_1^4 e^u\,du$$

$$2e^{\sqrt{x}}\Big|_1^4 =$$

$$2e^{\sqrt{4}} - 2e^{\sqrt{1}}$$

$$2e^2 - 2e$$

$f(?) = 2$ so $f^{-1}(2) = ?$

$f(1) = 2$ so $f^{-1}(2) = 1$

$g(x) = f^{-1}(x)$

4. Find $(f^{-1})'(a)$ for $f(x) = \dfrac{x+1}{2-x}$ at $a = 2$.

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$$f'(x) = \frac{(2-x)(1)-(x+1)(-1)}{(2-x)^2} = \frac{2-x+x+1}{(2-x)^2}$$

$$f'(x) = \frac{3}{(2-x)^2} = \frac{3}{(2-1)^2} = 3$$

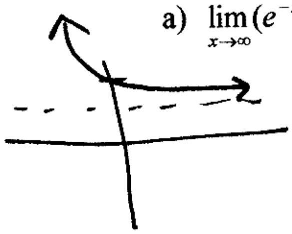
$$= \frac{1}{3} = 1/3$$

5. Find the limit.

a) $\lim\limits_{x\to\infty}(e^{-x}+1) =$ 1

b) $\lim\limits_{x\to\infty}\dfrac{e^x - e^{-x}}{e^x + e^{-x}} =$ 1

c) $\lim\limits_{x\to-\infty}(-e^{2x}-1) =$ -1



$$\lim_{x\to\infty} \frac{e^{2x}-1}{e^{2x}+1} = \frac{1-\frac{1}{e^{2x}}}{1+\frac{1}{e^{2x}}}$$

6. Show that the function $y = e^x + e^{-x/2}$ satisfies the differential equation $2y'' - y' - y = 0$

$$y' = e^x + e^{-x/2}(-1/2) = e^x - \tfrac{1}{2}e^{-x/2}$$

$$y'' = e^x - \tfrac{1}{2}\cdot e^{-x/2}(-1/2) = e^x + \tfrac{1}{4}e^{-x/2}$$

$$2(e^x + \tfrac{1}{4}e^{-x/2}) - (e^x - \tfrac{1}{2}e^{-x/2}) - (e^x + e^{-x/2})$$

$$2e^x + \tfrac{1}{2}e^{-x/2} - e^x + \tfrac{1}{2}e^{-x/2} - e^x - e^{-x/2} = 0$$

yes

7. Evaluate the integral:

a) $\int e^{-x}\sqrt{e^{-x}+3}\,dx$

b) $\int e^{3x}\sec(e^{3x})\tan(e^{3x})dx$

c) $\int_1^4 \dfrac{e^{\sqrt{x}}}{\sqrt{x}}dx$

$u = e^{\sqrt{x}}$

$du = \dfrac{1}{2\sqrt{x}}dx$

$u = e^{-x}+3$

$du = -e^{-x}dx$

$-\int u^{1/2}\,du$

$-\dfrac{1}{3/2}u^{3/2} + C$

$$\boxed{-\frac{2}{3}(e^{-x}+3)^{3/2} + C}$$

$u = e^{3x}$

$du = e^{3x}(3)dx$

$\frac{1}{3}\int \sec u \tan u\, du$

$$\frac{1}{3}\sec(e^{3x}) + C$$

$2\int_1^4 e^u\,du$

$2e^{\sqrt{x}}\Big|_1^4 =$

$2e^{\sqrt{4}} - 2e^{\sqrt{1}}$

$2e^2 - 2e$