

1. Find the Cartesian coordinates for the polar coordinate

a) $(3, 5\pi/4)$ $(-3\sqrt{2}/2, 3\sqrt{2}/2)$

$$x = r \cos \theta = 3 \cos 5\pi/4 = -3\sqrt{2}/2$$

$$y = r \sin \theta = 3 \sin 5\pi/4 = -3\sqrt{2}/2$$

b) $(3, -5\pi/3) = (3/2, 3\sqrt{3}/2)$

$$x = 3 \cos(-5\pi/3) = 3(1/2)$$

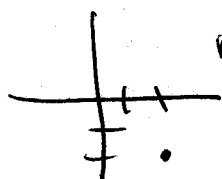
$$y = 3 \sin(-5\pi/3) = 3(\sqrt{3}/2)$$

and two different Polar coordinates for the points:

a) $(2, -2)$ $(2\sqrt{2}, 3\pi/4)$ and $(2\sqrt{2}, -\pi/4)$

$$r^2 = 4 + 4 = 8 \quad (-2\sqrt{2}, 3\pi/4)$$

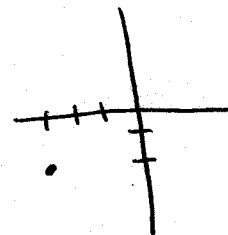
$$r = \sqrt{8} = 2\sqrt{2}$$



b) $(-3, -2)$ $(\sqrt{13}, \tan^{-1}(2/3) + \pi)$ and $(-\sqrt{13}, \tan^{-1}(2/3))$

$$r^2 = 9 + 4 = 13$$

$$r = \sqrt{13}$$



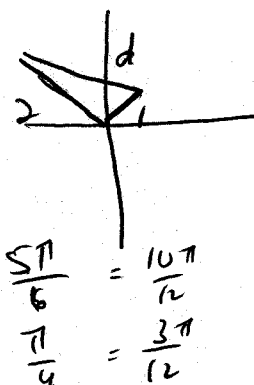
2. Find the distance between the two points with polar coordinates

$(2, 5\pi/6)$ and $(1, \pi/4)$

$$d^2 = 2^2 + 1^2 - 2(2)(1)\cos(5\pi/6 - \pi/4)$$

$$3 - 4 \cos 7\pi/12$$

$$d = \sqrt{3 - 4 \cos 7\pi/12}$$



3. Find the slope of the tangent line to the curve $r = \cos 2\theta$ at $\theta = \pi/4$

$$\frac{dr}{d\theta} = -2 \sin 2\theta$$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta} = \frac{-2 \sin 2\theta (\sin \theta) + \cos 2\theta (\cos \theta)}{-2 \sin 2\theta (\cos \theta) - \cos 2\theta (\sin \theta)}$$

$$\frac{dy}{dx} = \frac{-2 \sin \pi/2 \cdot \sin \pi/4 + \cos \pi/2 \cos \pi/4}{-2 \sin \pi/2 \cdot \cos \pi/4 - \cos \pi/2 \sin \pi/4} = \frac{-2(1)(\sqrt{2}/2) + 0}{-2(1)(\sqrt{2}/2) - 0} = 1$$

4. Find a polar equation for the curve represented by the Cartesian equation $x - 2y = 3$.

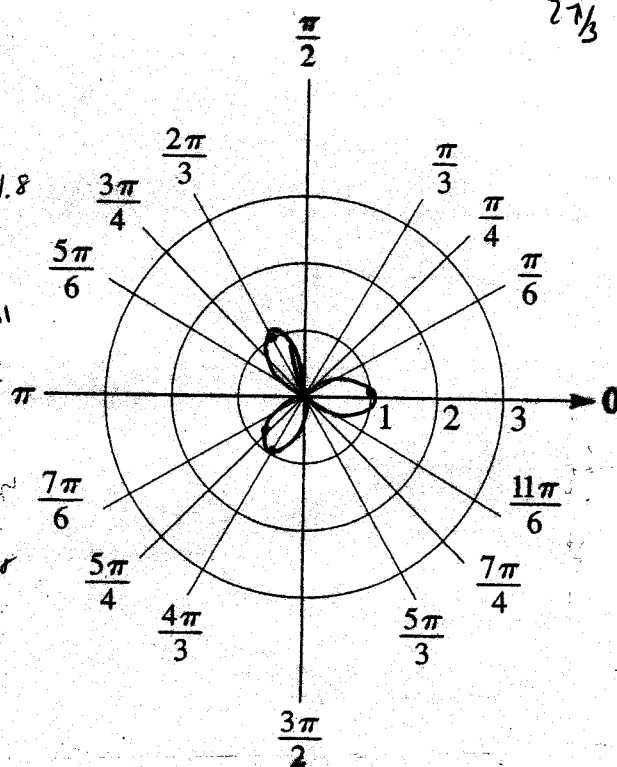
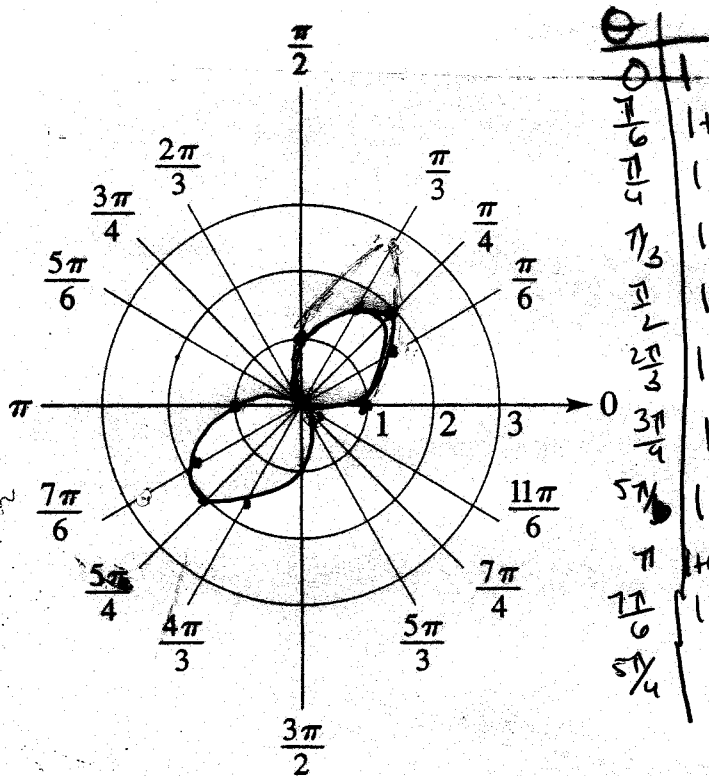
$$r \cos \theta - 2r \sin \theta = 3$$

$$r = \frac{3}{\cos \theta - 2 \sin \theta}$$

5. Sketch the curve with the given polar equation.

a) $r = 1 + \sin 2\theta$

b) $r = \cos 3\theta$



6. Find the points on the curve $r = 1 + \cos \theta$ where the tangent line is horizontal or vertical.

$$\frac{dy}{dx} = \frac{dr/d\theta \sin \theta + r \cos \theta}{dr/d\theta \cos \theta - r \sin \theta}$$

$$dr/d\theta = -\sin \theta$$

$$= \frac{-\sin^2 \theta + (1 + \cos \theta) \cos \theta}{-\sin \theta \cos \theta - (1 + \cos \theta) \sin \theta} = 0 \text{ horizontal}$$

$$= 0 \text{ vertical}$$

$$-\sin^2 \theta + \cos \theta + \cos^2 \theta = 0$$

$$-(1 - \cos^2 \theta) + \cos \theta + \cos^2 \theta = 0$$

$$-1 + \cos \theta + 2\cos^2 \theta = 0$$

$$(2\cos \theta - 1)(\cos \theta + 1) = 0$$

$$\cos \theta = 1/2 \quad \cos \theta = -1$$

$$\theta = \pi/3 + 2\pi k, \quad 5\pi/3 + 2\pi k \quad \theta = 3\pi/2 + 2\pi k$$

$$-\sin \theta \cos \theta - \sin \theta - \sin \theta \cos \theta = 0$$

$$-2\sin \theta \cos \theta - \sin \theta = 0$$

$$\sin \theta (-2\cos \theta - 1) = 0$$

$$\sin \theta = 0$$

$$\cos \theta = -1/2$$

$$\theta = \pi k$$

$$\theta = 2\pi/3 + 2\pi k, \quad 4\pi/3 + 2\pi k$$