MATH 152 | **QUIZ 10** | NAME Answers
Mrs. Bonny Tighe | 25 points | Monday 11/21/05
12.11-11.1 | Due Monday 11/28/05

1. Expand each of the following as a power series using the binomial series. State the radius and interval of convergence.

a) $(2-x^2)^{1/2}$ $\quad \sqrt{2}\left(1+\left(\frac{-x^2}{2}\right)\right)^{1/2}$ $\quad 1+\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{-x^2}{2}\right)^n \frac{(-1)^{n+1}(1\cdot3\cdot5\cdots(2n-3))}{n!\,2^n}$

$1+\frac{1}{2}+\frac{1}{2}\left(-\frac{1}{2}\right)+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)+\cdots$

$$1-\frac{x^2}{4}+\sum_{n=2}^{\infty}\frac{x^{2n}(1\cdot3\cdot5\cdots(2n-3))}{n!\,\cancel{2}\,2n}$$

$\left|\frac{x^2}{2}\right|<1$ $\quad R=\sqrt{2}$ $\quad I=[0,\sqrt{2}]$

$0<x^2<2$

$0<x<\sqrt{2}$

b) $\frac{x}{(2+x)^2}$ $\quad x(4)\left(1+\frac{x}{2}\right)^{-2}=4x\sum\left(\frac{x}{2}\right)^n\frac{(-1)^n(n+1)!}{n!}$

$1+-2+-2(-3)+(-2)(-3)(-4)+(-2)(-3)(-4)(-5)$

$$\sum_{n=0}^{\infty}\frac{x^{n+1}(-1)^n(n+1)}{2^{n-2}}$$

$\left|\frac{x}{2}\right|<1$ $\quad R=2$ $\quad I=[-2,2]$

c) $\frac{1}{\sqrt{4-x}}$

$\frac{1}{2\sqrt{1-\left(\frac{x}{4}\right)}}=\frac{1}{2}\left(1+\left(-\frac{x}{4}\right)\right)^{-1/2}=\frac{1}{2}\sum_{n=0}^{\infty}\frac{\left(\frac{-x}{4}\right)^n}{n!}\cdot\frac{(-1)^n(1\cdot3\cdot5\cdots(2n-1))}{2^n}$

$n=0 \quad n=1$

$1+\frac{-1}{2}+\frac{-1}{2}\cdot\frac{-3}{2}+\frac{-1}{2}\cdot\frac{-3}{2}\cdot\frac{-5}{2}+1\cdot3\cdot5\cdots(2n-1)$ $\quad \sum_{n=0}^{\infty}\frac{1}{2}\frac{x^n(1\cdot3\cdot5\cdots(2n-1))}{2^{2n}\cdot2^n\,n!}$

$\frac{(-1)^n(2n-1)\cdots1\cdot3\cdot5}{2^n}$

$$\sum_{n=0}^{\infty}\frac{x^n(1\cdot3\cdot5\cdots(2n-1))}{n!\cdot2^{3n+1}}$$

$\left|\frac{x}{4}\right|<1$ $\quad R=4$

$I=[-4,4]$

$t = \cos^{-1}(y/2)$

$x = 2\sin t \cos t$

$\frac{x}{\sin t} = 2\cos t$

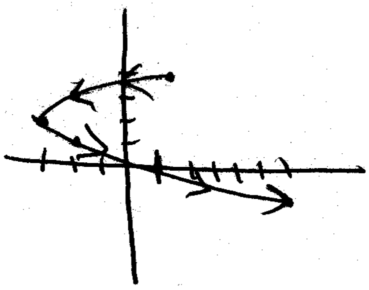
$\frac{x}{2\sin t} = \cos t$

$y = 2\left(\frac{x}{2\sin t}\right)$

2. Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases. Then eliminate the parameter to find a Cartesian equation of the curve.

a) $x = t^2 - 3,\ \ y = 2 - t,\ \ -2 \le t \le 3$

t	x	y
-2	1	4
-1	-2	3
0	-3	2
1	-2	1
2	1	0
3	6	-1



$t = 2 - y$

$x = (2-y)^2 - 3$

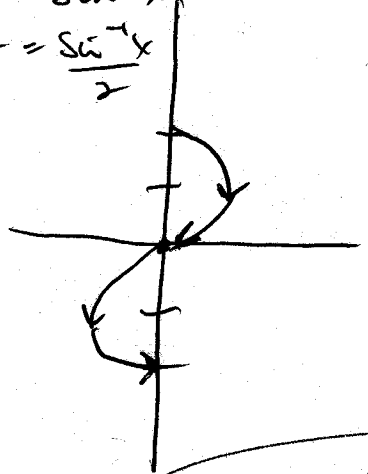
$\cancel{x = 4 - 4y + y^2 - 3}$ OR

$x = y^2 - 4y + 1$

b) $x = \sin\ 2t,\ \ y = 2\cos t,\ \ 0 \le t \le \pi$

$\cancel{2}t = \sin^{-1} x$

$t = \frac{\sin^{-1} x}{2}$

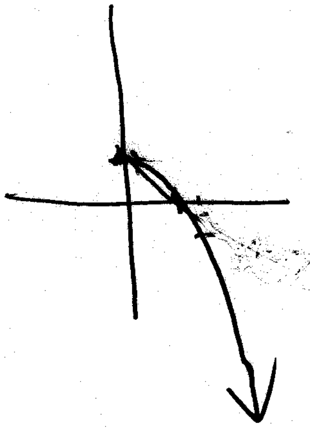


t	x	y
0	0	2
$\pi/6$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\pi/4$	1	$\sqrt{2}$
$\pi/3$	$\frac{\sqrt{3}}{2}$	1
$\pi/2$	0	0
$2\pi/3$		
$5\pi/4$		
$5\pi/6$		
π	0	-2

$y = 2\cos\left(\frac{\sin^{-1} x}{2}\right)$

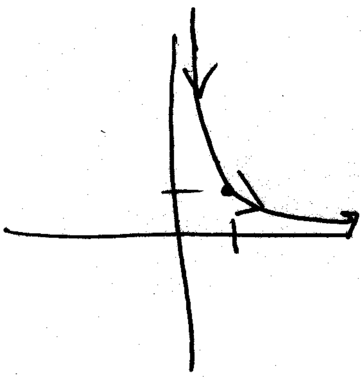
c) $x = t\sqrt{t},\ \ y = 1 - t^3$

t	x	y
*		
0	0	1
1	1	0
4	8	-63



$y = 1 - x^2$

d) $x = e^{2t},\ \ y = e^{-3t}$



t	x	y
-2		
-1	$\frac{1}{e^2}$	e^3
0	1	1
1	e^2	e^{-3}
2	e^4	e^{-6}
3	e^6	e^{-9}

$e^t = \sqrt{x}$

$y = \cancel{e}^{\cancel{3}}(\sqrt{x})^{-3}$

$y = x^{-3/2}$