MATH 152 | **QUIZ 1** | NAME Answers

Mrs. Bonny Tighe | 7.1-7.2 | Wednesday 9/7/05

25 points

1. Find $(f^{-1})'(a)$ for $f(x) = \dfrac{x+1}{2-x}$ at $a = 2$.

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))} = \frac{1}{3}$$

$f(?) = 2$ so $f'(2) = ?$

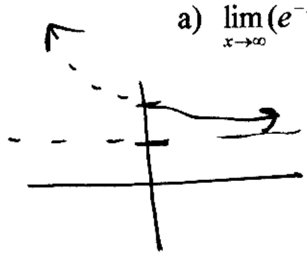
$\dfrac{x+1}{2-x} = 2 \qquad x+1 = -2x+4$

$3x = 3, \; x = 1$

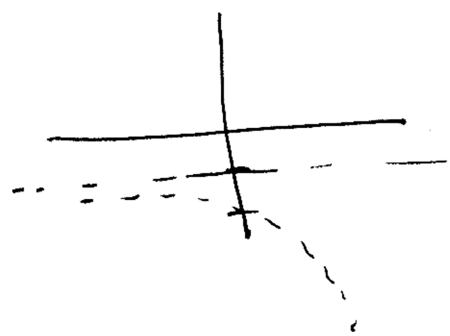
$$f'(x) = \frac{(2-x)(1) - (x+1)(-1)}{(2-x)^2} = \frac{2-x+x+1}{(2-x)^2} = \frac{3}{(2-x)^2}$$

2. Find the limit.

a) $\lim\limits_{x\to\infty}(e^{-x}+1) = 1$ b) $\lim\limits_{x\to\infty}\dfrac{e^x - e^{-x}}{e^x + e^{-x}} = 1$ c) $\lim\limits_{x\to-\infty}(-e^{2x}-1) = -1$



$$\lim_{x\to\infty} \frac{1 - \frac{1}{e^{2x}}}{1 + \frac{1}{e^{2x}}} = 1$$



3. Find dy/dx if $xy = y^3 + e^x e^y$

$$x\,dy/dx + y\cdot 1 = 3y^2\,dy/dx + e^x e^y\,dy/dx + e^y e^x$$

$$x\,dy/dx - 3y^2\,dy/dx - e^x e^y\,dy/dx = e^y e^x - y$$

$$dy/dx = \frac{e^y e^x - y}{x - 3y^2 - e^x e^y}$$

4. Find a formula for the inverse of the function $f(x) = \dfrac{x-1}{2x+3}$

$$x = \frac{y-1}{2y+3}$$

$$y = f^{-1}(x) = \frac{3x+1}{1-2x} \text{ or } \frac{-3x-1}{2x-1}$$

$2xy + 3x = y - 1$

$2xy - y = -3x - 1$

$y(2x-1) = -3x - 1$

$$2e^{x^{1/2}} \cdot x^{-1}$$

5. Find the derivative: a) $y = (2e^{\sqrt{x}})\left(\frac{1}{x}\right)$

$$dy/dx = (2e^{\sqrt{x}})(-1x^{-2}) + x^{-1}(2e^{\sqrt{x}})\left(\frac{1}{2\sqrt{x}}\right)$$

$$-\frac{2e^{\sqrt{x}}}{x^2} + \frac{e^{\sqrt{x}}}{x\sqrt{x}}$$

b) $g(x) = \dfrac{e^{\tan x}}{2+\sqrt{x}}$ $\quad g'(x) = \dfrac{(2+\sqrt{x})(e^{\tan x})\sec^2 x - e^{\tan x}\left(\frac{1}{2\sqrt{x}}\right)}{(2+\sqrt{x})^2}$

$$g'(x) = \frac{2\sqrt{x}(2+\sqrt{x})(e^{\tan x})\sec^2 x - e^{\tan x}}{2\sqrt{x}(2+\sqrt{x})^2}$$

6. Evaluate the integral:

a) $\int e^{-x}\sqrt{e^{-x}+3}\,dx$ $\qquad$ b) $\int e^{3x}\sec(e^{3x})\tan(e^{3x})dx$ $\qquad$ c) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ $\quad 2\int_1^4 e^u\,du$

$-\int u^{1/2}\,du$ $\quad u = e^{-x}+3$, $du = -e^{-x}dx$ $\quad \frac{1}{3}\int \sec u \tan u\,du$ $\quad u = e^{3x}$ $du = 3e^{3x}dx$ $\quad u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$ $\quad 2e^{\sqrt{x}}\Big|_1^4$

$-\frac{2}{3}(e^{-x}+3)^{3/2} + C$ $\qquad \frac{1}{3}\sec(e^{3x}) + C$ $\qquad 2e^2 - 2e^1$

7. Show that the function $y = e^x + e^{-x/2}$ satisfies the differential equation $2y'' - y' - y = 0$

$y' = e^x + e^{-x/2}(-1/2)$

$y'' = e^x - \frac{1}{2}e^{-x/2}(-1/2)$

$e^x + \frac{1}{4}e^{-x/2}$

$$2\left(e^x + \frac{1}{4}e^{-x/2}\right) - \left(e^x - \frac{1}{2}e^{-x/2}\right) - \left(e^x + e^{-x/2}\right)$$

$$2e^x + \frac{1}{2}e^{-x/2} - e^x + \frac{1}{2}e^{-x/2} - e^x - e^{-x/2}$$

$$= 0$$