13. Use the Limit Comparison Test to determine whether the series converges or diverges. $\sum_{n=2}^{\infty} \frac{2}{n^2 - \sqrt{n}}$

Compare: $b_n = \frac{1}{n^2}$, convergent by p-series, $p > 1$

$$\lim_{n\to\infty} \left| \frac{\frac{2}{n^2-\sqrt{n}}}{\frac{1}{n^2}} \right| = \lim_{n\to\infty} \left| \frac{2}{n^2-\sqrt{n}} \cdot \frac{n^2}{1} \right| = 2 > 0$$

both are convergent.

14. Use the ratio test to determine if the following series is absolutely convergent, conditionally convergent or divergent. ~~Find the radius of convergence and the interval of convergence if it is convergent.~~

$$\sum_{n=2}^{\infty} \frac{1\cdot 4\cdot 7\cdots(3n-5)}{(\ln n)n!}(-1)^{n-1}$$

$$\lim_{n\to\infty} \left| \frac{1\cdot 4\cdot 7\cdots(3n-5)(3n-2)}{\ln(n+1)\,(n+1)!} \cdot \frac{\ln(n)\; n!}{1\cdot 4\cdot 7\cdots(3n-5)} \right|$$

$$\lim_{n\to\infty} \left| \frac{\ln n}{\ln(n+1)} \cdot \frac{3n-2}{n+1} \right| = 3 > 1$$ so absolutely divergent

15. Find a power series representation for the function and determine the interval of convergence. a) $f(x)=\dfrac{3}{3+x^2}$

$$\frac{1}{1-\left(\frac{-x^2}{3}\right)} = \sum_{n=0}^{\infty}\left(\frac{-x^2}{3}\right)^n = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{3^n}$$

$$\lim_{n\to\infty}\left|\frac{x^{2n+2}}{3^{n+1}}\cdot\frac{3^n}{x^{2n}}\right| = \left|\frac{x^2}{3}\right| < 1 \quad [0,3) = I$$

test endpoints

$$\sum_{n=0}^{\infty}\frac{(-1)^n(0)}{3^n} = 0 \checkmark$$

$$\sum_{n=0}^{\infty}\frac{(3)^n}{3^n} = \phi \checkmark \text{ Dvg}$$

16. Evaluate the indefinite integral as an infinite series. $\int \frac{\sin 2x}{x}\,dx$

$$\int \frac{1}{x}\sum_{n=0}^{\infty}\frac{(-1)^n(2x)^{2n+1}}{(2n+1)!} = \int\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}\,2^{2n+1}}{(2n+1)!}$$

$$C + \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}\,2^{2n+1}}{(2n+1)!\,(2n+1)}$$

17. Find the Taylor series for f(x) centered at the given value of a and find its radius of convergence. $f(x)=\sqrt{x+2}$, *centered at* $a=2$

$f'(x)=\frac{1}{2}(x+2)^{-1/2}$ | $\frac{1}{2\sqrt{4}}$ | $\frac{1}{4}$ | $\frac{1}{2^2}$

$f''(x)=-\frac{1}{4}(x+2)^{-3/2}$ | $\frac{-1}{4(4)^{3/2}}$ | $\frac{-1}{4(8)}$ | $\frac{-1}{2^5}$

$f'''(x)=\frac{3}{8}(x+2)^{-5/2}$ | $\frac{3}{8}\cdot\frac{1}{4^{5/2}}$ | $\frac{3}{8(32)}$ | $\frac{3}{2^8}$

$f^{(4)}(x)=\frac{-15}{16}(x+2)^{-7/2}$ | $\frac{-15}{16}\cdot\frac{1}{4^{7/2}}$ | $\frac{-15}{16(2^7)}$ | $\frac{-15}{2^{11}}$

at a=2

$$\boxed{1+\sum_{n=\phi}^{\infty}(-1)^n\frac{(1\cdot3\cdot5\ldots(2n-1))(x-2)^n}{n!\,2^{3n-1}}}$$

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}(1\cdot3\cdot5\ldots(2n-1)(2n+1))}{(n+1)!\,2^{3n+2}}\cdot\frac{n!\,2^{3n-1}}{(x-2)^n(1\cdot3\cdot5\ldots(2n-1))}\right|=\lim_{n\to\infty}\left|\frac{(2n+1)(x-2)}{(n+1)(8)}\right|=$$

$\left|\frac{1}{8}(2)(x-2)\right|<1$

$\left|\frac{1}{4}x-2\right|<1$ $R=4$

18. Expand $\frac{1}{(1-2x)^3}$ as a power series using the binomial series. State the radius of convergence.

$(1-2x)^{-3}$

$(1+(-2x))^{-3}$

$$\frac{(n+2)!}{2}\cdot\frac{(-2x)^n}{n!}=\frac{(-1)^n2^nx^n(-1)^n(n+2)!}{2n!}$$

$$\frac{1}{2}+\frac{2-3}{2}+\frac{2(-3)(-4)}{2}+\frac{2(-3)(-4)(-5)}{2}+\frac{2(-3)(-4)(-5)(-6)}{2}+\ldots$$

n=0, n=1, n=2, n=3, n=4

$$\boxed{\sum_{n=0}^{\infty}x^n2^{n-1}(n+1)(n+2)}$$

$|2x|<1$

$R=1/2$

$$\frac{dy}{dt} = \frac{(1+t)(1) - t(1)}{(1+t)^2} = \frac{1}{(1+t)^2} \qquad dy/dt = \frac{1}{1+t}$$

19. Find the length of the curve, $x = \frac{t}{1+t}$, $y = \ln(1+t)$, $0 \le t \le 2$

$$\int_0^2 \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt = \int_0^2 \sqrt{\frac{1}{(1+t)^4} + \frac{1}{(1+t)^2}}\, dt$$

$$\int_0^2 \frac{\sqrt{1+(1+t)^2}}{(1+t)^2}\, dt \qquad \begin{aligned} 1+t &= \tan\theta \\ dt &= \sec^2\theta\, d\theta \end{aligned}$$

$$\int_0^2 \frac{\sqrt{1+\tan^2\theta}}{\tan^2\theta}(\sec^2\theta\, d\theta)$$

$$\int_0^2 \frac{\sec^3\theta}{\tan^2\theta}\, d\theta = \int_0^2 \sec\theta \csc^2\theta\, d\theta$$

20. a) Find a Cartesian equation for the curve described by the polar equation
$r = \sec\theta$

$r\cos\theta = 1$

$x = 1$

~~b) Find a~~ polar equation for the curve $x = -y^2$

$r\cos\theta = -r^2\sin^2\theta$

$\cos\theta = -r\sin^2\theta$

$$r = \frac{\sin^2\theta}{\cos\theta} \text{ or } \sec\theta\tan\theta$$

c) Find the slope of the tangent line to the polar curve $r = \ln\theta$, at $\theta = e$.

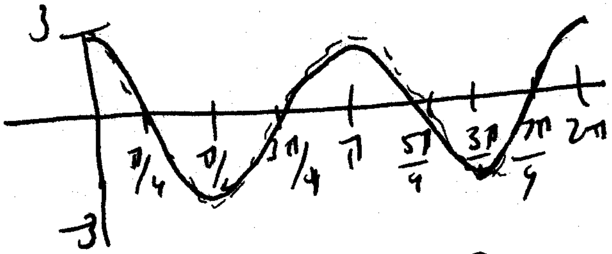
$$dy/dx = \frac{dr/d\theta \sin\theta + r\cos\theta}{dr/d\theta \cos\theta - r\sin\theta} = \frac{\frac{1}{\theta}(\sin\theta) + \ln\theta\cos\theta}{\frac{1}{\theta}\cos\theta - \ln\theta\sin\theta} =$$

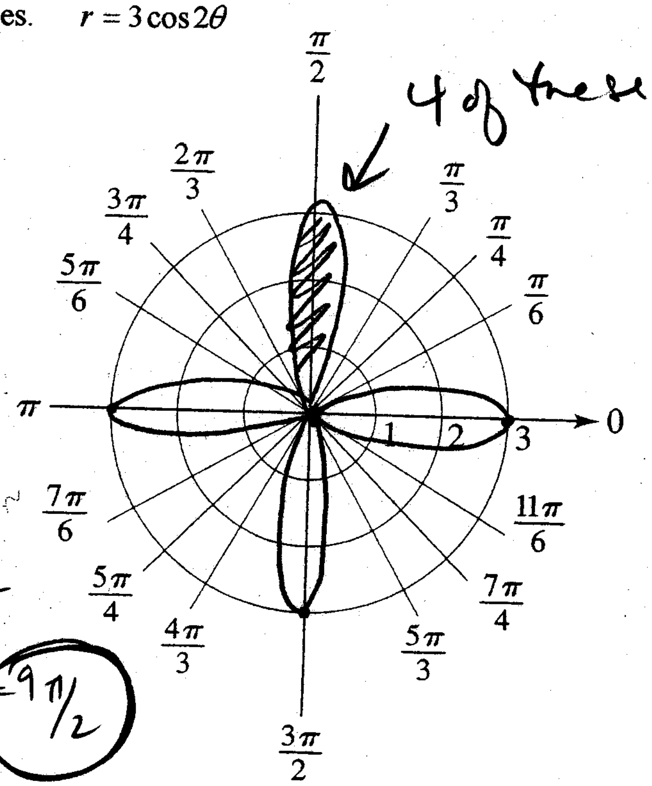
$$\frac{\frac{1}{e}\sin e + \ln e\cos e}{\frac{1}{e}\cos e - \ln e\sin e} \qquad \frac{\sin e + e\cos e}{\cos e - e\sin e} = dy/dx$$

$$18\int_{\pi/4}^{3\pi/4}\left(\frac{1}{2} + \frac{1}{2}\cos 4\theta\right)d\theta = 18\left(\frac{1}{2}\theta + \frac{1}{8}\sin 4\theta\right)\Big|_{\pi/4}^{3\pi/4}$$

9

21. Sketch the curve and find the area that it encloses. $r = 3\cos 2\theta$





$$9\theta + \frac{9}{4}\sin 4\theta \Big|_{\pi/4}^{3\pi/4}$$

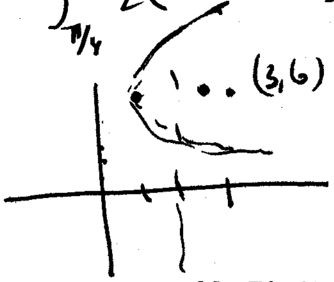
$A = \frac{1}{2} r^2 d\theta$

$$= \left(9(3\pi/4) + \cancel{\tfrac{9}{4}\sin 3\pi}\right) - \left(9(\pi/4) + \cancel{\tfrac{9}{4}\sin \pi}\right)$$

$$4\int_{\pi/4}^{3\pi/4} \frac{1}{2}(9\cos^2 2\theta)\,d\theta$$

$$\frac{27\pi}{4} - \frac{9\pi}{4}$$

$$\frac{18\pi}{4} = \frac{9\pi}{2}$$



$p = 1/2$

$4p = 2$

22. Find an equation for the conic which satisfies the given conditions.

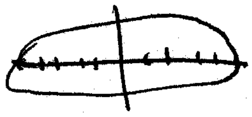
a) Parabola, directrix x = 2, focus (3,6)

$y^2 = 2x$ $\quad$ $y^2 = 2x$

b) Ellipse, foci ($\pm$2,0), vertices ($\pm$5, 0)

$2^2 = 5^2 - b^2$

$b^2 = 25 - 4$



$$\boxed{\frac{x^2}{25} + \frac{y^2}{21} = 1}$$

c) Find the vertices, foci and asymptotes of the hyperbola and sketch its graph.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

vertices: $(\pm 3, 0)$

foci: $(\pm 5, 0)$

asymptotes: $y = \pm \frac{4}{3}x$

$c^2 = 16 + 9 = 25$

$c = 5$