MATH 152 **FINAL EXAM** NAME Answers

Mrs. Bonny Tighe 200 points

Section ____ Fri 12/16/05

22 problems with 10 points each

1. Find dy/dx. a) $3e^{y} + \ln xy = \sin^{-1} x$

$\ln x + \ln y$

$$3e^{y}\,dy/dx + \frac{1}{x} + \frac{1}{y}\,dy/dx = \frac{1}{\sqrt{1-x^2}}$$

[multiply both sides by $xy\sqrt{1-x^2}$]

$$3e^{y}xy\sqrt{1-x^2}\,dy/dx + y\sqrt{1-x^2} + x\sqrt{1-x^2}\,dy/dx = xy$$

$$dy/dx = \frac{xy - y\sqrt{1-x^2}}{3e^{y}xy + x\sqrt{1-x^2}}$$

b) $y = e^{\sin^{-1} x}$

$$dy/dx = e^{\sin^{-1} x}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

2. Evaluate: a) $\int \frac{\sec^2\alpha}{2+\tan\alpha}\,d\alpha =$ ______ b) $\int \frac{e^{-x}}{1-2e^{-x}}\,dx =$ ______

$$\int \frac{1}{u}\,du \qquad u = 2+\tan\alpha \qquad du = \sec^2\alpha\,d\alpha$$

$$\ln u + C$$

$$\ln|2+\tan\alpha| + C$$

$$\frac{1}{2}\int \frac{1}{u}\,du \qquad u = 1-2e^{-x} \qquad du = 2e^{-x}\,dx$$

$$\frac{1}{2}\ln|u| + C$$

$$\frac{1}{2}\ln|1-2e^{-x}| + C$$

3. Evaluate: $\int_1^4 \underbrace{\ln t^4}_{u}(\underbrace{\sqrt{t})dt}_{dv} =$ ________

$du = \frac{1}{t^4}(4t^3) \quad v = \frac{1}{3/2}t^{3/2}$

$\int_1^4 \ln t^4 \sqrt{t}\,dt = \ln t^4\left(\frac{2}{3}t^{3/2}\right) - \int \frac{2}{3}t^{3/2}\cdot\frac{4}{t}\,dt =$

$= \ln t^4\left(\frac{2}{3}t^{3/2}\right) - \frac{8}{3}\int t^{1/2}dt =$

$= \ln t^4\left(\frac{2}{3}t^{3/2}\right) - \frac{8}{3}\cdot\frac{2}{3}t^{3/2} \Big|_1^4$

$\left(4\ln 4\left(\frac{2}{3}(8)\right) - \frac{16}{9}(8)\right) - \left(4\ln(1)\left(\frac{2}{3}\right)(1) - \frac{16}{9}(1)\right)$

$\frac{64}{3}\ln 4 - \frac{128}{9} + \frac{16}{9} = \frac{64}{3}\ln 4 - \frac{112}{9}$

4. Find the following limits. a) $\lim_{x\to\infty}\left(\frac{e^{2x}}{x^3}\right) =$ $\infty$

$\frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty} \frac{2e^{2x}}{3x^2} = \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty} \frac{4e^{2x}}{6x} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to\infty} \frac{8e^{2x}}{6} = \infty$

b) $\lim_{x\to 0^+}(\cos x)^{1/x^2} =$ $e^{-1/2}$

c) $\lim_{x\to 0^+}\sin x\ln x =$ $0$

$\ln y \; \frac{\ln \cos x}{x^2} = \frac{0}{0} \overset{}{=} L'H$

$= \frac{\frac{1}{\cos x}(-\sin x)}{2x} = \frac{-\tan x}{2x} = \frac{0}{0}$

$\frac{-\sec^2 x}{2} = -\frac{1}{2}$

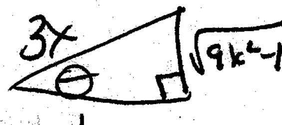
$\ln y = -1/2$ so $y = e^{-1/2}$

$\lim_{x\to 0^+} \frac{\ln x}{\csc x} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc x\cot x}$

$\lim_{x\to 0^+}\left(\frac{1}{-x\csc x\cot x}\right)$

$= \frac{-\sin x \tan x}{x} = 0$




5. Evaluate: $\int x^3\sqrt{9x^2-1}\,dx$

$x = \frac{1}{3}\sec\theta$

$\int \frac{1}{27}\sec^3\theta\sqrt{\sec^2\theta-1}\left(\frac{1}{3}\sec\theta\tan\theta\,d\theta\right)$ $dx = \frac{1}{3}\sec\theta\tan\theta\,d\theta$

$\frac{1}{81}\int \sec^4\theta\tan^2\theta\,d\theta = \frac{1}{81}\int \tan^2\theta(1+\tan^2\theta)\sec^2\theta\,d\theta$

$\frac{1}{81}\int \tan^2\theta\sec^2\theta\,d\theta + \frac{1}{81}\int \tan^4\theta\sec^2\theta\,d\theta$

$\frac{1}{81}\left(\frac{1}{3}\tan^3\theta\right) + \frac{1}{81}\left(\frac{1}{5}\tan^5\theta\right) + C =$

$\frac{1}{243}\left(\sqrt{9x^2-1}\right)^3 + \frac{1}{405}\left(\sqrt{9x^2-1}\right)^5 + C$

6. Evaluate the integral using partial fractions: $\int \frac{x+3}{x^2-2x-3}dx =$ ______

$\frac{A}{x-3} + \frac{B}{x+1} = \frac{x+3}{x^2-2x-3}$ $(x-3)(x+1)$

$A(x+1) + B(x-3) = x+3$

$(x=-1)$ $-4B = 2$, $B = -\frac{1}{2}$

$(x=3)$ $4A = 6$ $A = \frac{3}{2}$

$\frac{3}{2}\int \frac{1}{x-3}dx - \frac{1}{2}\int \frac{1}{x+1}dx$

$\frac{3}{2}\ln|x-3| - \frac{1}{2}\ln|x+1| + C$

7. Evaluate $\int \sin^5 x \cos^3 x \, dx =$ ______________

$\int \sin^5 x(\cos^2 x)\cos x\,dx$
$(1-\sin^2 x)$

$\int \sin^5 x - \sin^7 x\,(\cos x\,dx)$

$\boxed{\frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C}$

8. Find the Macluarin series for the given function and find the radius and interval of convergence. $f(x)=\dfrac{1}{x-1}$ $\quad (x-1)^{-1}$

$f'(x) = -1(x-1)^{-2} = \frac{-1}{(-1)^2} = -1$

$f''(x) = 2(x-1)^{-3} = \frac{2}{(-1)^3} = -2$

$f'''(x) = -6(x-1)^{-4} = \frac{-6}{(-1)^4} = -6$

$f''''(x) = 4!(x-1)^{-5} = \frac{24}{(-1)^5} = -24$

$-\dfrac{n!\,x^n}{n!}$

$\sum_{n=0}^{\infty} -x^n$

$\boxed{R=1 \quad I=(-1,1)}$

$\dfrac{-1}{1-x}$

$\lim_{n\to\infty}\left|\dfrac{-x^{n+1}}{-x^n}\right| = |x|<1$

9. Use substitution to evaluate: $\int \frac{dx}{e^x - e^{-x}}$ $\left(\frac{e^x}{e^x}\right)$

$u = e^x$

$du = e^x\,dx$

$\frac{du}{u} = dx$

$\int \frac{e^x\,dx}{e^{2x}-1} = \int \frac{\cancel{u}\,\frac{du}{\cancel{u}}}{u^2-1} = \int \frac{du}{(u+1)(u-1)}$

$\frac{A}{u+1} + \frac{B}{u-1} = \frac{1}{u^2-1}$

$A(u-1) + B(u+1) = 1$

$(u=1)$ $B = 1/2$

$(u=-1)$ $A = -1/2$

$-\frac{1}{2}\int \frac{1}{u+1}\,du + \frac{1}{2}\int \frac{1}{u-1}\,du$

$-\frac{1}{2}\ln|u+1| + \frac{1}{2}\ln|u-1| + C$

$-\frac{1}{2}\ln|e^x+1| + \frac{1}{2}\ln|e^x-1| + C$

or $\ln\sqrt{\frac{e^x-1}{e^x+1}} + C$

10. Determine whether each integral is divergent or convergent and evaluate those that are convergent using improper integrals.

a) $\int_0^\infty \frac{2}{(x+1)^2}\,dx$

$\lim_{t\to\infty} \int_0^t 2(x+1)^{-2}\,dx$

$\lim_{t\to\infty} \frac{2}{-1}(x+1)^{-1}\Big|_0^t =$

$\lim_{t\to\infty}\left[\frac{-2}{t+1} - \frac{-2}{0+1}\right]$

$0 + 2 = 2$

Convergent

b) $\int_0^3 \frac{1}{x^2\sqrt{x}}\,dx$ =

$\lim_{t\to 0} \int_t^3 x^{-5/2}\,dx =$

$\lim_{t\to 0} \frac{1}{-3/2} x^{-3/2}\Big|_t^3 = \frac{-2}{3} x^{-3/2}\Big|_t^3$

$\lim_{t\to 0}\left[\frac{-2}{3(3^{3/2})} - \left(\frac{-2}{3t^{3/2}}\right)\right] =$

$\lim_{t\to 0}$ $+\infty$ $= \infty$

Divergent

$\frac{1}{2}\int_0^2 \sec^2\theta\csc\theta\, d\theta = \csc\theta\tan\theta + \int \tan\theta\csc\theta\cot\theta\, d\theta =$

11. Find the length of the curve. $x = y^3,\ 0 \le y \le 2$

$\frac{dx}{dy} = 3y^2$

$L = \int_0^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\int_0^2 \sqrt{1 + 9y^4}\, dy$

$y^2 = \frac{1}{3}\tan\theta$

$2y\, dy = \frac{1}{3}\sec^2\theta\, d\theta$

$dy = \frac{1}{6}\frac{\sec^2\theta}{\frac{1}{3}\tan\theta}\, d\theta$

$\int_0^2 \sqrt{1 + \tan^2\theta}\left(\frac{1}{2}\frac{\sec^2\theta}{\tan\theta}\right) d\theta$

$\int_0^2 \frac{1}{2}\frac{\sec^3\theta}{\tan\theta}\, d\theta = \frac{1}{2}\int_0^2 \sec^2\theta\csc\theta\, d\theta =$

$dv \quad u$

$v = \tan\theta \quad du = -\csc\theta\cot\theta$

12. Determine whether the series are convergent or divergent using the appropriate test, and if they are convergent, find its sum.

a) $\sum_{n=2}^{\infty} \frac{2}{n^2 - 1} = \frac{2}{(n+1)(n-1)}$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!}$

$\boxed{\text{Sum} = \cos 3}$

$\boxed{\text{Limit Comparison Test}}$

$\boxed{\text{Ratio Test}}$

$\sum_{n=2}^{\infty}\left(\frac{1}{n-1} - \frac{1}{n+1}\right) = \frac{1}{1} + \frac{1}{2} + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}}$

$-\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} - \frac{1}{6}$

$\lim_{n\to\infty}\left|\frac{\frac{2}{n^2-1}}{\frac{1}{n^2}}\right| =$

$\lim_{n\to\infty}\left|\frac{3^{2n+2}}{(2n+2)(2n)!}\cdot\frac{(2n)!}{3^{2n}}\right| = 0 < 1$

$\lim_{n\to\infty}\left|\frac{2}{n^2-1}\cdot\frac{n^2}{1}\right| = 2 > 0$

so convergent

so both are convergent

Sum $= 3/2$