MATH 152 **EXAM IIA** NAME Answers

Mrs. Bonny Tighe 8.3 – 12.3

100 points SECTION _______ Wed. 10/26/05

1. Evaluate: $\int \sin x(e^x)dx$

$u = \sin x \quad dv = e^x$

$du = \cos x \quad v = e^x$

$\int \sin x\, e^x = \sin x\, e^x - \int e^x \cos x\, dx$

$v = e^x \quad u = \cos x$

$dv = e^x \quad du = -\sin x$

$\int \sin x\, e^x = \sin x\, e^x - \left[\cos x\, e^x - \int e^x(-\sin x)dx\right] = \sin x\, e^x - \cos x\, e^x - \int e^x \sin x\, dx$

$2\int \sin x\, e^x = \sin x\, e^x - \cos x\, e^x$

$= \frac{1}{2} e^x(\sin x - \cos x) + C$

2. Evaluate: $\int \frac{e^{2x}-1}{e^{2x}-e^x-6}\, dx$

$u = e^x \qquad du = e^x dx$

$du/u = dx$

$\int \frac{u^2-1}{u^2-u-6} \cdot \left(\frac{du}{u}\right) = \int \frac{u^2-1}{u(u-3)(u+2)} du$

$\frac{A}{u} + \frac{B}{u-3} + \frac{C}{u+2} = \frac{u^2-1}{u(u-3)(u+2)}$

$A(u-3)(u+2) + Bu(u+2) + Cu(u-3) = u^2-1$

$u=0 \qquad -6A = -1 \qquad A = 1/6$

$u=3 \qquad 15B = 8 \qquad B = 8/15$

$u=-2 \qquad 10C = 3 \qquad C = 3/10$

$\frac{1}{6}\int \frac{1}{u} du + \frac{8}{15}\int \frac{1}{u-3} du + \frac{3}{10}\int \frac{1}{u+2} du = \frac{1}{6}\ln|u| + \frac{8}{15}\ln|u-3| + \frac{3}{10}\ln|u+2| + c$

$\frac{1}{6}\ln e^x + \frac{8}{15}\ln|e^x-3| + \frac{3}{10}\ln|e^x+2| + c$

$\frac{1}{6}x + \frac{8}{15}\ln|e^x-3| + \frac{3}{10}\ln|e^x+2| + C$

$x = 2\tan\theta$

3. Evaluate: $\int \frac{\sqrt{x^2+4}}{x^2}dx$

$dx = 2\sec^2\theta\, d\theta$

$$\int \frac{\sqrt{4\tan^2\theta+4}}{4\tan^2\theta}(2\sec^2\theta\, d\theta) = \int \frac{\cancel{2}\sec\theta \cdot 2\sec^2\theta\, d\theta}{\cancel{4}\tan^2\theta} =$$

$$\int \sec^2\theta \cdot \csc\theta\, d\theta = \csc\theta\tan\theta - \int \cancel{\tan}\theta(-\csc\theta\cancel{\cot}\theta)\,d\theta$$

$dv \quad u$

$v = \tan\theta \quad du = -\csc\theta\cot\theta$

$$= \csc\theta\tan\theta + \int\csc\theta\, d\theta = \csc\theta\tan\theta + \ln|\csc\theta - \cot\theta| + C$$

$$\frac{\sqrt{x^2+4}}{x}\left(\frac{x}{2}\right) + \ln\left|\frac{\sqrt{x^2+4}}{x} - \frac{2}{x}\right| + C$$

4. a) Determine the values of x for which the series is convergent. $\sum_{n=1}^{\infty}\frac{x^n}{4^n}$

$|x| < 4$

$-4 < x < 4$

b) Define each of the following:

i) a divergent sequence ______________________

ii) a monotonic series always increasing or always decreasing

iii) a geometric series the summation of a sequence in which you find the next term by multiplying by the same ratio.

5. Evaluate: $\int \tan^3 2x \sec^2 2x \, dx$

$$\int \tan 2x \, (\tan^2 2x) \sec^2 2x \, dx = \int \sec 2x (\sec^2 2x - 1) \quad \sec 2x \tan 2x \, dx$$
$$(\sec^2 2x - 1)$$

$$= \int \sec^3 2x \;\sec 2x \tan 2x \, dx \;-\; \int \sec 2x \,(\sec 2x \tan 2x)\, dx$$

$$\frac{1}{2} \cdot \frac{1}{4} \sec^4 2x - \frac{1}{2} \cdot \frac{1}{2} \sec^2 2x \quad + C$$

$$\frac{1}{8} \sec^4 2x - \frac{1}{4} \sec^2 2x + C$$

$$\text{or } \frac{1}{8} \tan^4 2x + C$$

6. Determine whether each integral is divergent or convergent and evaluate those that are convergent.

a) $\int_{-1}^{3} \frac{1}{(x+1)^2} dx$ $\quad \lim_{t \to -1} \int_t^3 (x+1)^{-2} dx =$

$$\lim_{t \to -1} \frac{1}{-1}(x+1)^{-1} \Big|_t^3 = \left(\frac{-1}{3+1}\right) - \left(-\frac{1}{t+1}\right)$$

$$\lim_{t \to -1} \left(\frac{-1}{4} + \frac{1}{t+1}\right) \quad \text{Divergent}$$

b) $\int_1^{\infty} \frac{\ln x}{x^2} dx$ $\quad \lim_{t \to \infty} \int_1^t \cancel{\ln x \left(\frac{1}{x}\right) \cdot \frac{1}{x}} dx \quad \int_1^t \underset{u}{\ln x} \; \underset{dv}{x^{-2}}$

$$du = \frac{1}{x} \quad v = \frac{1}{-1} x^{-1}$$

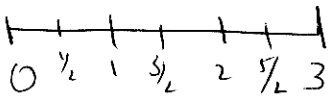
$$\lim_{t \to \infty} \int_1^t \left[\ln x \left(-\frac{1}{x}\right) - \int -\frac{1}{x} \cdot \frac{1}{x} dx \right] =$$

L'H

$$\lim_{t \to \infty} -\frac{\ln x}{x} + \frac{1}{-1} x^{-1} \Big|_1^t = \lim_{t \to \infty} \frac{-\frac{1}{x}}{1} - \frac{1}{x} = -0 - 0 = 0 + 1$$

Convergent

7. Use Simpson's Rule to approximate the area under the curve $f(x) = \dfrac{1}{x^2+1}$ from $x = 0$ *to* $x = 3$ with n = 6.



$$\Delta x = \frac{3-0}{6} = \frac{1}{2}$$

$$S_6 = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4 \cdots + 4f(x_{n-1}) + f(x_n) \right]$$

$$S_6 = \frac{1}{6}\left[\frac{1}{0+1} + 4\left(\frac{1}{1/4+1}\right) + 2\left(\frac{1}{1+1}\right) + 4\left(\frac{1}{9/4+1}\right) + 2\left(\frac{1}{4+1}\right) + 4\left(\frac{1}{\frac{25}{4}+1}\right) + \frac{1}{9+1} \right]$$

$$S_6 = \frac{1}{6}\left[1 + 4\left(\frac{4}{5}\right) + 2\left(\frac{1}{2}\right) + 4\left(\frac{4}{13}\right) + 2\left(\frac{1}{5}\right) + 4\left(\frac{4}{29}\right) + \frac{1}{10} \right]$$

8. Determine whether the sequence is convergent or divergent. If it converges, find the limit.

a) $\{2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}, \ldots\}$ $\quad a_n = \frac{n+1}{n^2}$

$$\lim_{n \to \infty} \frac{n+1}{n^2} = 0$$ convergent

b) $a_n = \ln(n+1) - \ln(n-1)$ $= \quad a_n = \ln\left(\frac{n+1}{n-1}\right)$

$$\lim_{n \to \infty} \ln\left(\frac{n+1}{n-1}\right) = \ln(1) = 0$$ convergent

9. Find the sum for each of the following series: a) $\sum_{n=1}^{\infty} 4^{-n}$

$$\sum_{n=1}^{\infty} \frac{1}{4^n} = \frac{1}{4}\left(\frac{1}{4}\right)^{n-1} \qquad Sum = \frac{a}{1-r}$$

$$\frac{\frac{1}{4}}{1 - 1/4} = \frac{1}{4-1} = 1/3$$

b) $\sum_{n=1}^{\infty} \frac{2}{n^2 + 3n + 2}$

$$(n+1)(n+2)$$

$$\frac{A}{n+1} + \frac{B}{n+2} = \frac{2}{(n+2)(n+1)}$$

$$A(n+2) + B(n+1) = 2$$

$(n = -1)$ $\quad A = 2$

$(n = -2)$ $\quad -B = 2$

$$\sum_{n=1}^{\infty} \left(\frac{2}{n+1} + \frac{-2}{n+2}\right) =$$

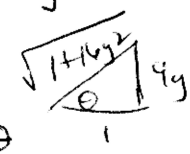
$$\frac{2}{2} + \frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \frac{2}{6} \ldots$$

$$- \frac{2}{3} - \frac{2}{4} - \frac{2}{5} - \frac{2}{6} \ldots$$

$$1$$

10. Find the surface area when the curve $x = 1 + 2y^2$ on the interval $1 \le y \le 2$ is rotated about the x-axis.

$$\int_1^2 2\pi y \sqrt{1 + (dx/dy)^2}\, dy \qquad dx/dy = 4y$$



$$\int_1^2 2\pi y \sqrt{1 + 16y^2}\, dy \qquad y = \tfrac{1}{4}\tan\theta \qquad dy = \tfrac{1}{4}\sec^2\theta\, d\theta$$

$$2\pi \int_1^2 \tfrac{1}{4}\tan\theta \sqrt{1+\tan^2\theta}\,\left(\tfrac{1}{4}\sec^2\theta\right) d\theta$$

$$\frac{2\pi}{16} \int \tan\theta\,(\sec^3\theta)\, d\theta$$

$$-\frac{\pi}{8} \int \frac{-\sin\theta}{\cos^4\theta}\, d\theta = -\frac{\pi}{8}\,\frac{1}{-3}\cos^{-3}\theta \Big|_1^2 = \frac{\pi}{24}\cdot\frac{1}{\cos^3\theta}\Big|_1^2 = \frac{\pi}{24}\left(\sqrt{1+16y^2}\right)^3 \Big|_1^2 =$$

$$= \boxed{\frac{\pi}{24}\left(\sqrt{65}\right)^3 - \frac{\pi}{24}\left(\sqrt{17}\right)^3} \text{ or } = \frac{\pi}{24}\left[65\sqrt{65} - 17\sqrt{17}\right]$$

11. Use the integral test to determine if each series is convergent or divergent.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 - 4}$ Can't use not ~~continuous~~ decreasing - yes positive - yes

$$\lim_{t\to\infty} \int_1^t \frac{1}{(x+2)(x-2)}\, dx$$

$$A(x-2) + B(x+2) = 1$$

$$4B = 1 \quad B = 1/4$$

$$-4A = 1 \quad A = -1/4$$

$$\lim_{t\to\infty} \int_1^t -\tfrac{1}{4}\cdot\frac{1}{x+2} + \tfrac{1}{4}\,\frac{1}{x-2}\, dx$$

$$\lim_{t\to\infty} -\tfrac{1}{4}\ln|x+2| + \tfrac{1}{4}\ln|x-2| \Big|_1^t = \lim_{t\to\infty} \ln\sqrt[4]{\frac{x-2}{x+2}}\Big|_1^t$$

$$\lim_{t\to\infty} \ln\sqrt[4]{\frac{\cancel{t}-2}{\cancel{t}+2}} - \ln\sqrt[4]{\frac{3}{3}}$$

$$\ln\sqrt[4]{1} - \ln\sqrt[4]{1/3} = -\ln\sqrt[4]{1/3}$$

Convergent

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln n}$

$$\lim_{t\to\infty} \int_1^t \frac{1}{\ln n}\cdot\frac{1}{n}\, dn$$

$$\lim_{t\to\infty} \ln|\ln n| \Big|_1^t = \infty$$

$$\ln|\ln t| - \ln|\ln 1|$$

∞ und

divergent