MATH 152
Mrs. Bonny Tighe

EXAM II

8.3 – 12.3
100 points

NAME Answers

SECTION _______ Wed. 10/26/05

1. Evaluate: $\int e^{-x}\cos x \, dx$ $= e^{-x}\sin x + \int \sin x \,(e^{-x})\,dx$

$u = e^{-x}$, $du = -e^{-x}$ dv, $v = \sin x$ dv, $v = -\cos x$ u, $du = -e^{-x}dx$

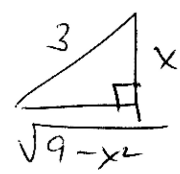
$$\int e^{-x}\cos x\,dx = e^{-x}\sin x + e^{-x}(-\cos x) + \int \cos x\,(-e^{-x})\,dx$$

$$2\int e^{-x}\cos x\,dx = e^{-x}\sin x - \cos x\, e^{-x}$$

$$= \tfrac{1}{2}e^{-x}(\sin x - \cos x) + C$$

2. Evaluate: $\int \frac{\sqrt{9-x^2}}{x}dx$

$x = 3\sin\theta$
$dx = 3\cos\theta\, d\theta$



$$\int \frac{\sqrt{9-9\sin^2\theta}}{\cancel{3}\sin\theta}\cancel{3}\cos\theta\, d\theta = 3\int \frac{\cos^2\theta}{\sin\theta}d\theta = 3\int \frac{1-\sin^2\theta}{\sin\theta}d\theta$$

$$3\int \frac{1}{\sin\theta}d\theta - \int \sin\theta\, d\theta$$

$$3\int \csc\theta\, d\theta + \cos\theta$$

$$3\ln|\csc\theta - \cot\theta| + \cos\theta + C$$

$$3\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + (3)\frac{\sqrt{9-x^2}}{3} + C$$

3. a) Determine the values of x for which the series is convergent. $\sum_{n=1}^{\infty}(x-3)^n$

$|x-3| < 1 \qquad -1 < x-3 < 1 \qquad$ so $2 < x < 4$

b) Define each of the following:

i) a monotonic sequence____

ii) A convergent sequence______________________________________

iii) a Geometric series____

4. Evaluate: $\int \sin^3 2x \cos^3 2x \, dx$

$\int \sin^3 2x \cos^2 2x \cos 2x \, dx$ $\quad (1-\sin^2 2x)$

$\int \sin^3 2x \cos 2x \, dx - \int \sin^5 2x \cos 2x \, dx$

$\frac{1}{2} \cdot \frac{1}{4} \sin^4 2x - \frac{1}{2} \cdot \frac{1}{6} \sin^6 2x + C$

$\frac{1}{8} \sin^4 2x - \frac{1}{12} \sin^6 2x + C$

or $\int \cos^3 2x \sin^2 2x \sin 2x \, dx$ $\quad (1-\cos^2 2x)$

$\int \cos^3 2x - \cos^5 2x \, (\sin 2x \, dx)$

$-\frac{1}{2} \cdot \frac{1}{4} \cos^4 2x + \frac{1}{2} \cdot \frac{1}{6} \cos^6 2x$

or $-\frac{1}{8} \cos^4 2x + \frac{1}{12} \cos^6 2x + C$

/

5. Evaluate: $\int \frac{e^{2x}+1}{e^{2x}-e^{x}-6}\,dx$

$u = e^x$

$du = e^x\,dx$

$\frac{du}{u} = dx$

$\int \frac{u^2+1}{u^2-u-6}\cdot\frac{du}{u} = \int \frac{u^2+1}{u(u-3)(u+2)}\,du = \frac{1}{-6}\int \frac{1}{u}\,du + \frac{2}{3}\int \frac{1}{u-3}\,du + \frac{1}{2}\int \frac{1}{u+2}\,du$

$A(u-3)(u+2) + B(u)(u+2) + C(u)(u-3) = u^2+1$

$(u=3)$ $15B = 10$, $B = 2/3$

$(u=-2)$ $10C = 5$, $C = 1/2$

$(u=0)$ $-6A = 1$, $A = -1/6$

$-6\ln u + \frac{2}{3}\ln|u-3| + \frac{1}{2}\ln|u+2|$

$-6\ln e^x + 2/3\ln|e^x-3| + \frac{1}{2}\ln|e^x+2|$

$-\frac{1x}{6} + 2/3\ln|e^x-3| + \frac{1}{2}\ln|e^x+2| + C$

6. Determine whether each integral is divergent or convergent and evaluate those that are convergent.

a) $\int_1^\infty \frac{\ln x}{x}\,dx$

Type I

$\lim_{t\to\infty}\int_1^t (\ln x)\cdot\frac{1}{x}\,dx = \lim_{t\to\infty}\frac{1}{2}(\ln x)^2\Big|_1^t$

$\lim_{t\to\infty}\left(\frac{1}{2}(\ln t)^2 - \frac{1}{2}\ln(1)^2\right) = \lim_{t\to\infty}\frac{1}{2}(\ln t)^2 = \infty$

so divergent

b) $\int_{-1}^3 \frac{2}{(x+1)^2}\,dx$

Type II

$\lim_{t\to -1}\int_t^3 2(x+1)^{-2}\,dx =$

$\lim_{t\to -1}\frac{2}{-1}(x+1)^{-1}\Big|_t^3 = \lim_{t\to -1}\frac{-2}{(x+1)}\Big|_t^3 =$

$\lim_{t\to -1}\left[\frac{-2}{4} - \left(\frac{-2}{t+1}\right)\right] = \lim_{t\to -1}\left[-\frac{1}{4} + \frac{2}{t+1}\right] =$ undefined

so divergent

7. Determine whether the sequence is convergent or divergent. If it converges, find the limit.

a) $\{\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \dots\}$ $a_n = \frac{n}{n+1}$ Convergent

$\frac{1}{3}\ \frac{2}{4}\ \frac{3}{5}\ \frac{4}{6}\ \frac{5}{7}$

$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)$ 1

b) $a_n = \ln(n-1) - \ln(n+1)$ $\lim_{n\to\infty} \ln\left|\frac{n-1}{n+1}\right| \cdot$ ~~lim~~

$\ln(1) = 0$ So convergent

8. Find the sum for each of the following convergent series.

b) $\sum_{n=1}^{\infty} \frac{3}{5^n}$ $= \sum_{n=1} 3\left(\frac{1}{5}\right)^n = \sum_{n=1}^{\infty} \frac{3}{5}\left(\frac{1}{5}\right)^{n-1}$ $\text{Sum} = \frac{a}{1-r}$

$\text{Sum} = \frac{3/5}{1-1/5} = \frac{3}{4}$

c) $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$ $\frac{A}{n+1} + \frac{B}{n+2}$

$A(n+2) + B(n+1) = 1$

$(n=-1)$ $A = 1$

$(n=-2)$ $-B = 1$

$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$

$-\frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \frac{1}{6}$

$1/2$ Sum

9. Use the Integral Test to determine whether the series is convergent or divergent.

a) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ decreasing ✓ positive ✓ continuous ✓

b) $\sum_{n=1}^{\infty} ne^{-n}$ decreasing ✓ positive ✓ continuous ✓

$\lim_{n\to\infty} \int_1^n \frac{1}{\ln x} \cdot \frac{1}{x} dx$

$\lim_{n\to\infty} \ln|\ln x| \Big|_1^n$

$\lim_{n\to\infty} \ln|\ln n| - \ln|\ln 1|$

$\ln|\infty) - \ln|0|$

∞ - undefined

so divergent

$\lim_{t\to\infty} \int_1^t n e^{-n} dn$, $u = n$, $du = dn$, dv, $v = -e^{-n}$

$\lim_{t\to\infty} n(-e^{-n}) - \int_1^t -e^{-n} dn$

$\lim_{t\to\infty} -ne^{-n} - e^{-n} \Big|_1^t$

$(-te^{-t} - e^{-t}) - (-e^{-1} - e^{-1})$

$\lim_{t\to\infty}$ $0 - 0 + 2e^{-1} = 2/e^1$

so Convergent

10. Find the length of the arc of the curve $y = \frac{x^2}{2} - \frac{\ln x}{4}$ on the interval $2 \le x \le 4$

$L = \int_2^4 \sqrt{1 + (dy/dx)^2}\, dx$

$dy/dx = \frac{1}{2}(2x) - \frac{1}{4}\left(\frac{1}{x}\right)$

$= \left(x - \frac{1}{4x}\right)^2$

$x^2 - 2(x)\left(\frac{1}{4x}\right) + \frac{1}{16x^2}$

$x^2 - \frac{1}{2} + \frac{1}{16x^2}$

$L = \int_2^4 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}}\, dx$

$\int_2^4 \sqrt{\left(x + \frac{1}{4x}\right)^2}\, dx$

$\int_2^4 x + \frac{1}{4x}\, dx = \frac{1}{2}x^2 + \frac{1}{4}\ln x \Big|_2^4 =$

$\left(\frac{1}{2}(16) + \frac{1}{4}\ln(4)\right) - \left(\frac{1}{2}(2)^2 + \frac{1}{4}\ln 2\right)$

$8 + \frac{1}{4}\ln 4 - \frac{1}{2}(4) - \frac{1}{4}\ln 2$

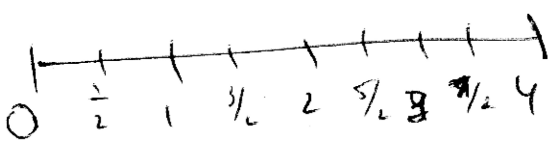
$6 + \frac{1}{4}\ln 4 - \frac{1}{4}\ln 2$

11. Use Simpson's Rule to approximate the area under the curve $f(x) = xe^x$ from $x = 0$ *to* $x = 4$ with n = 8.

$$\Delta x = \frac{4-0}{8} = \frac{1}{2}$$

$$S_8 = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n) \right]$$

$$S_8 = \frac{1}{6}\left[f(0) + 4f(1/2) + 2f(1) + 4f(3/2) + 2f(2) + 4f(5/2) + 2f(3) + 4f(7/2) + f(4) \right]$$



$$S_8 = \frac{1}{6}\left[0 + 4\left(\tfrac{1}{2}e^{1/2}\right) + 2(1)e^1 + 4\left(\tfrac{3}{2}\right)e^{3/2} + 2(2)e^2 + 4\left(\tfrac{5}{2}\right)e^{5/2} + 2(3)e^3 + 4\left(\tfrac{7}{2}\right)e^{7/2} + 4e^4 \right]$$

$$S_8 = \frac{1}{6}\left[2e^{1/2} + 2e + 6e^{3/2} + 4e^2 + 10e^{5/2} + 6e^3 + 14e^{7/2} + 4e^4 \right]$$