MATH 152 **EXAM IA** NAME Answers

Mrs. Bonny Tighe 7.1-8.2

100 points SECTION _______ Wed 9/28/05

There are 11 problems with 10 points each

1. Find the numerical value of each expression.

a) $\cosh^{-1}(1) =$ 0 b) $\sinh(\ln 3) =$ $\frac{3}{2} - \frac{1}{6}$ = 4/3 c) $\log_3 27\sqrt{3} =$ $3\frac{1}{2}$

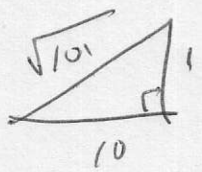
$\frac{e^x + e^{-x}}{2} = 1$ $e^x + e^{-x} = 2$ $\frac{e^{\ln 3} - e^{-\ln 3}}{2}$ $\log_3 3^3 3^{1/2}$

$e^{2x} + 1 = 2e^x$ $e^x = 1$

$e^{2x} - 2e^x + 1 = 0$ $(e^x - 1)^2 = 0$ $\frac{3 - \frac{1}{3}}{2}$

d) $e^{(2\ln 3 + \ln 2)} =$ 18 e) $\sin(\arctan(0.1)) =$ $1/\sqrt{101}$

$e^{\ln 9 + \ln 2} = e^{\ln 18}$



2. Find dy/dx: a) y = ln(ln(csc *x*))

$dy/dx = \frac{1}{\ln(\csc x)} \cdot \frac{1}{\csc x} \cdot (-\csc x \cot x) = \frac{-\cot x}{\ln(\csc x)}$

b) $\cos^3(xy) = 2e^y$

$3\cos^2 xy(-\sin xy)(x\,dy/dx + y) = 2e^y\,dy/dx$

$(-\sin xy)\,3x\cos^2 xy\,dy/dx + 3y\cos^2 xy(-\sin xy) = 2e^y\,dy/dx$

$(-\sin xy)\,3y\cos^2 xy = 2e^y\,dy/dx - 3x\cos^2 xy\,dy/dx(-\sin xy)$

$\frac{-3y\cos^2 xy \sin xy}{2e^y + 3x\cos^2 xy \sin xy} = dy/dx$

3. Find the equation of the tangent line to the curve $y = (\cancel{\sin}^{-1}\sqrt{x})(5^{x^3})$ at the point $(1, 5\pi/\cancel{2}4)$.

$(\tan^{-1}\sqrt{x})(5^{x^3})$

$$m = dy/dx = (\tan^{-1}\sqrt{x})(5^{x^3}(3x^2)\ln 5) + 5^{x^3}\left(\frac{1}{1+x}\right)\left(\frac{1}{2\sqrt{x}}\right)$$

$$= \tan^{-1}(1)(5^1)(3)(1)\ln 5 + 5^1\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)$$

$$= \underbrace{\frac{\pi}{4}(15)\ln 5 + \frac{5}{4}}$$

$y - y_1 = m(x - x_1)$

$y - 5\pi/4 = \cancel{m}(x-1)$

$$y - \frac{5\pi}{4} = \frac{15\pi\ln 5 + 5}{4}(x-1)$$

4. Evaluate: a) $\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx =$ ________

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\int 2^u\,du$

$$\frac{2\cdot 2^u}{\ln 2} = \frac{2\cdot 2^{\sqrt{x}}}{\ln 2}\Big|_1^4 = \frac{2\cdot 2^2}{\ln 2} - \frac{2\cdot 2^1}{\ln 2} = \frac{8-4}{\ln 2} = \frac{4}{\ln 2}$$

b) $\int \frac{1}{x\ln x}\,dx =$ $\ln(\ln x) + C$

$\int \frac{1}{u}\,du$

$u = \ln x$

$du = \frac{1}{x}\,dx$

5. Find f '(x): $f(x) = \ln \sqrt{\dfrac{2x^2-1}{\tan x}}$

$f'(x) = \frac{1}{2}\ln(2x^2-1) - \frac{1}{2}\ln(\tan x)$

$f'(x) = \frac{1}{2} \cdot \frac{1}{2x^2-1}(4x) - \frac{1}{2}\frac{1}{\tan x}(\sec^2 x)$

$f'(x) = \frac{2x}{2x^2-1} - \frac{\sec^2 x}{2\tan x}$

6. Evaluate using integration by parts: $\int u\,dv = uv - \int v\,du$

a) $\int \tan^{-1} t\, dt =$ __________

u dv

$du = \frac{1}{1+t^2}$ $v = t$

$\int \tan^{-1} t\,dt = \tan^{-1} t\,(t) - \frac{1}{2}\int 2t\left(\frac{1}{1+t^2}\right)dt$

$= t\tan^{-1} t - \frac{1}{2}\ln|1+t^2| + C$

b) $\int_0^1 e^x \sin x\, dx =$ __________

u dv

$du = e^x$ $v = -\cos x$

$\int_0^1 e^x \sin x\,dx = e^x(-\cos x) - \int -\cos x\, e^x\,dx = -e^x\cos x + \int \cos x\, e^x\,dx$

dv u

$v = \sin x$ $du = e^x$

$= -e^x\cos x + e^x \sin x - \int \sin x\, e^x\,dx$

$2\int_0^1 e^x \sin x\,dx = -e^x\cos x + e^x \sin x$

$\int e^x \sin x\,dx = \frac{1}{2}(-e^x\cos x + e^x \sin x)\Big|_0^1 =$

$\frac{1}{2}(-e^1\cos(1) + e^1\sin(1)) - \left(\frac{1}{2}(-e^0\cos 0 + e^0\sin 0)\right) =$

$-\frac{1}{2}e\cos(1) + \frac{1}{2}e\sin(1) + 1/2 - 0$

7. Evaluate: $\int \frac{1-x}{9+x^2}dx = $ ____________

$$\int \frac{1}{9+x^2}dx + \int \frac{-x}{9+x^2}dx$$

$$3\int \frac{1/9}{1+\left(\frac{x}{3}\right)^2}\,\frac{1}{3}dx \quad -\frac{1}{2}\int \frac{1}{u}du$$

$$\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}\ln|9+x^2| + C$$

8. Find the following limits. Use L'Hospital's Rule where appropriate.

a) $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2+x}\right) = $ 0

$$\frac{0}{0} \overset{L'H}{=} \lim_{x\to 0}\frac{\sin x}{2x+1} = \frac{0}{1} = 0$$

b) $\lim_{x\to\infty}\left(\frac{2x+2}{2x-1}\right)^x = $ 0

$$\ln y = x\ln\left(\frac{2x+2}{2x-1}\right) = \lim_{x\to\infty}\frac{\ln\left(\frac{2x+2}{2x-1}\right)}{\frac{1}{x}} = \frac{0}{0}$$

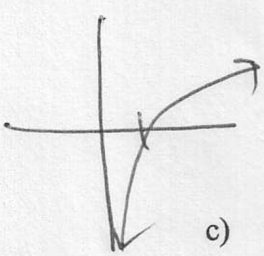
$$\overset{L'H}{=} \lim_{x\to\infty}\left(\frac{\frac{1}{2x+2}(2) - \frac{1}{2x-1}(2)}{-1/x^2}\right) =$$

$$\lim_{x\to\infty}\left(\frac{-2x^2}{2x+2} + \frac{2x^2}{2x-1}\right) =$$

$$\lim_{x\to\infty}\frac{-4x^3+2x^2+4x^3+4x^2}{(2x+2)(2x-1)}$$

$$\lim_{x\to\infty}\left(\frac{6x^2}{4x^2+\dots}\right) = 6/4 = 3/2$$

$\ln y = 3/2$ so $y = e^{3/2}$



c) $\lim_{x\to 0^+}\sqrt{x}\ln x = $ 0

$0\cdot\infty$

$$\lim_{x\to 0^+}\left(\frac{\ln x}{\frac{1}{\sqrt{x}}}\right) = \frac{\infty}{\infty} \overset{L'H}{=}$$

$$\lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}\right)\left(\frac{-2x^{3/2}}{1}\right)$$

$$\lim_{x\to 0^+}(-2\sqrt{x}) = 0$$

$$\int_0^{\pi/6} \sin^2 2x\,(\cos 2x)^{1/2}\,(\sin 2x\,dx)$$

9. Evaluate the integral. $\int_0^{\frac{\pi}{6}} \sin^3 2x\sqrt{\cos 2x}\; dx$

$$\int_0^{\pi/6} (1-\cos^2 2x)(\cos 2x)^{1/2}\sin 2x\,dx =$$

$$\int_0^{\pi/6} (\cos 2x)^{1/2}\sin 2x\,dx - \int_0^{\pi/6} (\cos 2x)^{5/2}\sin 2x\,dx$$

$u = \cos 2x$

$du = -\sin 2x(2)\,dx$

$$-\frac{1}{2}\int_0^{\pi/6} u^{1/2}\,du + \frac{1}{2}\int_0^{\pi/6} u^{5/2}\,du$$

$$-\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + \frac{1}{2}\cdot\frac{2}{7}u^{7/2}\Big|_0^{\pi/6} = -\tfrac{1}{3}(\cos 2x)^{3/2} + \tfrac{1}{7}(\cos 2x)^{7/2}\Big|_0^{\pi/6} =$$

10. Evaluate: a) $\int \frac{\operatorname{csc}h^2 x}{1-\coth x}dx$ = ____________

$u = 1-\coth x$

$du = -\operatorname{csch}^2 x\,dx$

$$-\int \frac{1}{u}\,du$$

$$-\ln|1-\coth x| + C$$

b) $\int e^{x^2}\cos(e^{x^2})2x dx$ = ____________

$u = e^{x^2}$

$du = e^{x^2}(2x)\,dx$

$$\int \cos u\,du$$

$\sin u + C$

$\sin(e^{x^2}) + C$

11. Use logarithmic differentiation to find the derivative for each of the following:

a) $f(x)=\sqrt{3e^x+2}\,(\sqrt{x}-\cos x)^5$ b) $y=(\sec x+2x)^{\sqrt{x}}$ (see below)

$\ln y = \frac{1}{2}\ln(3e^x+2)+5\ln(\sqrt{x}-\cos x)$

$\frac{1}{y}\,dy/dx = \frac{1}{2}\cdot\frac{1}{3e^x+2}(3e^x) + 5\cdot\frac{1}{\sqrt{x}-\cos x}\left(\frac{1}{2\sqrt{x}}+\sin x\right)$

$\frac{1}{y}\,dy/dx = \left(\frac{3e^x}{2(3e^x+2)} + \left(\frac{5}{\sqrt{x}-\cos x}\right)\left(\frac{1}{2\sqrt{x}}+\sin x\right)\right)\left(\sqrt{3e^x+2}\right)\left(\sqrt{x}-\cos x\right)^5$

b) $\ln y = \sqrt{x}\,\ln(\sec x+2x)$

$\frac{1}{y}\,dy/dx = \sqrt{x}\cdot\frac{1}{\sec x+2x}(\sec x\tan x+2) + \ln(\sec x+2x)\left(\frac{1}{2\sqrt{x}}\right)$

$dy/dx = \left(\frac{\sqrt{x}(\sec x\tan x+2)}{\sec x+2x} + \frac{\ln|\sec x+2x|}{2\sqrt{x}}\right)(\sec x+2x)^{\sqrt{x}}$