MATH 152 **EXAM I** NAME Answers

Mrs. Bonny Tighe 7.1-8.2

100 points SECTION ______ Wed 9/28/05

There are 11 problems with 10 points each

1. Find f '(x): $f(x) = \ln\sqrt{\dfrac{2x-1}{\sin x}}$ $= \frac{1}{2}\ln(2x-1) - \frac{1}{2}\ln(\sin x)$

$$f'(x) = \frac{1}{2}\cdot\frac{1}{2x-1}(2) - \frac{1}{2}\cdot\frac{1}{\sin x}(\cos x)$$

$$= \frac{1}{2x-1} - \frac{1}{2}\frac{\cos x}{\sin x} \quad \text{or} \quad \frac{1}{2x-1} - \frac{1}{2}\cot x$$

2. Find the equation of the tangent line to the curve $y = (\tan^{-1} x)(5^{x^3})$ at the point $(1, 5\pi/4)$.

$$y - y_1 = m(x - x_1)$$

$$y - 5\pi/4 = \Delta \quad (x-1)$$

$$y - 5\pi/4 = \left(\frac{\pi(15)\ln 5}{4} + 5/2\right)(x-1)$$

$$m = dy/dx = (\tan^{-1}x)(5^{x^3})(3x^2)\ln 5 + 5^{x^3}\left(\frac{1}{1+x^2}\right)$$

at $x = 1$

$$\tan^{-1}(1)(5^1)(3)(1)^2\ln 5 + 5^{(1)}\left(\frac{1}{1+1}\right)$$

$$m = \underbrace{\pi/4(15)\ln 5 + 5/2}$$

3. Find dy/dx: a) $y = \ln(\ln(\sec x))$

$dy/dx = \frac{1}{\ln(\sec x)} \cdot \frac{1}{\sec x} \cdot \sec x \tan x = \frac{1}{\ln(\sec x)} \cdot \tan x$

b) $\tan^3(xy) = 2e^y$ $\quad 3\tan^2(xy)[\sec^2 xy](x\, dy/dx + y) = 2e^y\, dy/dx$

$3x\tan^2 xy \sec^2 xy\, dy/dx + 3y\tan^2 xy \sec^2(xy) = 2e^y\, dy/dx$

$3y\tan^2 xy = 2e^y\, dy/dx - 3x\tan^2 xy \sec^2 xy\, dy/dx$

$\frac{3y\tan^2 xy \sec^2 xy}{2e^y - 3x\tan^2 xy \sec^2 xy} = dy/dx$

4. Evaluate: a) $\int_0^1 x^2 e^{-x^3}\, dx =$ ________

$-\frac{1}{3}\int e^u\, du$ $\qquad u = -x^3$

$du = -3x^2 dx$

$-\frac{1}{3}e^u \Big|_0^1 = -\frac{1}{3}e^{-x^3}\Big|_0^1 = -\frac{1}{3}e^{-1} - \left(-\frac{1}{3}e^0\right) = \frac{-1}{3e} + \frac{1}{3}$

b) $\int \frac{\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}\, dx =$ ________

$u = \sqrt{x}$

$2\int \sec u \tan u\, du$ $\qquad du = \frac{1}{2\sqrt{x}}\, dx$

$2\sec\sqrt{x} + C$

5. Evaluate: $\int \frac{1+x}{4-x^2}dx =$ ________

$\int \frac{1}{4-x^2}dx + \int \frac{x}{4-x^2}dx \quad -\frac{1}{2}\int \frac{1}{u}du$

$u = 4-x^2$

$du = -2x\,dx$

$2\int \frac{1/4}{1-\left(\frac{x}{2}\right)^2}\left(\frac{1}{2}dx\right) \quad -\frac{1}{2}\ln|4-x^2| + C$

$\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right) + -\frac{1}{2}\ln|4-x^2| + C$

6. Evaluate using integration by parts: $\int u\,dv = uv - \int v\,du$

a) $\int \cos^{-1} t\, dt =$ ________

$u \quad dv$

$du = \frac{-1}{\sqrt{1-t^2}} \quad v = t$

$\cos^{-1}t\,(t) - \int t\left(\frac{-1}{\sqrt{1-t^2}}\right)dt$

$t\cos^{-1}t + \frac{-1}{2}\int (1-t^2)^{-1/2}(-2)t\,dt$

$t\cos^{-1}t - \frac{1}{2}\cdot\frac{1}{1/2}(1-t^2)^{1/2} + C$

$t\cos^{-1}t - \sqrt{1-t^2} + C$

b) $\int_0^1 e^x \cos x\, dx =$ ________ $= (\cos x)e^x - \int e^x(-\sin x)\,dx$

$dv \quad u$

$v = e^x \quad du = -\sin x$

$dv \quad u$

$v = e^x \quad du = \cos x\,dx$

$\int e^x \cos x\,dx = (\cos x)e^x + \sin x(e^x) - \int e^x \cos x\,dx$

$= \frac{1}{2}(\cos x)e^x + \frac{1}{2}(\sin x)(e^x)\Big|_0^1 =$

$\left(\frac{1}{2}\cos(1)e + \frac{1}{2}\sin(1)e\right) - \left(\frac{1}{2}\cos(0)e^0 + \frac{1}{2}\sin(0)e^0\right) =$

$\frac{e\cos(1)}{2} + \frac{e\sin(1)}{2} - \frac{1}{2}$

7. Find the following limits. Use L'Hospital's Rule where appropriate.

a) $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2+x}\right) = \underline{0}$

$\frac{0}{0} \overset{L'H}{=} \lim_{x\to 0} \frac{\sin x}{2x+1} = \frac{0}{1} = 0$

b) $\lim_{x\to\infty}\left(\frac{x-2}{x+1}\right)^x = \underline{e^{-3}}$

$\ln y = \lim_{x\to\infty} x \ln\left(\frac{x-2}{x+1}\right)$

$\lim_{x\to\infty}\left(\frac{\ln\left(\frac{x-2}{x+1}\right)}{\frac{1}{x}}\right) = \frac{0}{0} \overset{L'H}{=}$

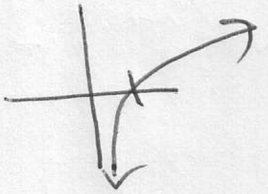
$\lim_{x\to\infty}\left(\frac{\frac{1}{x-2}-\frac{1}{x+1}}{-\frac{1}{x^2}}\right) =$

$\lim_{x\to\infty}\left(\frac{-x^2}{x-2}-\frac{-x^2}{x+1}\right) =$

$\lim_{x\to\infty}\left(\frac{\cancel{-x^3}-x^2+\cancel{x^3}-2x^2}{(x-2)(x+1)}\right)$

$\lim_{x\to\infty}\left(\frac{-3x^2}{x^2-x-2}\right) = -3$

$\ln y = -3$ so $y = e^{-3}$



c) $\lim_{x\to 0^+}\sqrt{x}\ln x = \underline{0}$

$0\cdot\infty$

$\lim_{x\to 0^+}\left(\frac{\ln x}{\frac{1}{\sqrt{x}}}\right) = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}\cdot\frac{-2x^{3/2}}{1}\right) = 0$

8. Find the numerical value of each expression.

a) $\sinh^{-1}(0) = \underline{0}$

$\frac{e^0-e^{-x}}{2} = 0 \quad e^x - e^{-x} = 0$

b) $\cosh(\ln 3) = \underline{\frac{3+\frac{1}{3}}{2} = 5/3}$

$\frac{e^{\ln 3}+e^{-\ln 3}}{2}$

c) $\log_2 4\sqrt{2} = \underline{5/2}$

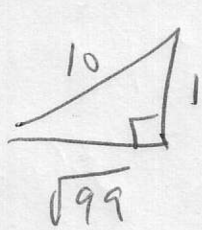
$2^2\cdot 2^{1/2}$

d) $e^{(\ln 3+2\ln 2)} = \underline{12}$

$e^{\ln 3+\ln 4} = e^{\ln(12)}$

$e^{2x}-2e^x-1=0$

e) $\tan(\arcsin(0.1)) = \underline{1/\sqrt{99}}$



9. Evaluate: a) $\int \frac{\operatorname{sech}^2 x}{1-\tanh x}dx$ = ____________

$u = 1-\tanh x$

$du = -\operatorname{sech}^2 x\, dx$

$-\int \frac{1}{u}\,du = -\ln|1-\tanh x| + C$

b) $\int \frac{\sin(\ln x)}{x}dx = -\cos(\ln x) + C$

$u = \ln x$

$du = \frac{1}{x}dx$

$\int \sin u\, du = -\cos u + C$

10. Use logarithmic differentiation to find the derivative for each of the following:

a) $f(x) = \left(\frac{x+2}{\cos x}\right)^4$

$\ln y = 4\ln(x+2) - 4\ln(\cos x)$

$\frac{1}{y}\,dy/dx = \frac{4}{x+2} - \frac{4}{\cos x}(\sin x)$

$\frac{dy}{dx} = \left(\frac{4}{x+2} - 4\tan x\right)\left(\frac{x+2}{\cos x}\right)^4$

b) $y = (\tan x + 2)^{\sqrt{x}}$

$\ln y = \sqrt{x}\,\ln(\tan x + 2)$

$\frac{1}{y}\,dy/dx = \sqrt{x}\left(\frac{\sec^2 x}{\tan x + 2}\right) + \ln(\tan x + 2)\left(\frac{1}{2\sqrt{x}}\right)$

$dy/dx = \left(\left(\frac{\sqrt{x}\sec^2 x}{\tan x + 2}\right) + \left(\frac{\ln(\tan x + 2)}{2\sqrt{x}}\right)\right)\left((\tan x + 2)^{\sqrt{x}}\right)$

11. Evaluate the integral. $\int_0^{\frac{\pi}{6}} \cos^3 2x\sqrt{\sin 2x}\, dx$

$\int_0^{\pi/6} \cos^2 2x\, (\sin 2x)^{1/2} \cos 2x\, dx$

$\int_0^{\pi/6} (1-\sin^2 2x)(\sin 2x)^{1/2} \cos 2x\, dx$

$\int_0^{\pi/6} \left((\sin 2x)^{1/2} - \sin^2 2x (\sin 2x)^{1/2}\right) \cos 2x\, dx$

$\int_0^{\pi/6} (\sin 2x)^{1/2} \cos 2x\, dx - \int_0^{\pi/6} (\sin 2x)^{5/2} \cos 2x\, dx$

$\frac{1}{2}\int u^{1/2}\, du - \frac{1}{2}\int u^{5/2}\, du =$

$\frac{1}{2} \cdot \frac{1}{3/2} u^{3/2} - \frac{1}{2} \cdot \frac{1}{7/2} \cdot u^{7/2} \Big|_0^{\pi/6} =$

$\frac{1}{3}(\sin 2x)^{3/2} - \frac{1}{7}(\sin 2x)^{7/2} \Big|_0^{\pi/6} = \left(\frac{1}{3}(\sin \pi/3)^{3/2} - \frac{1}{7}(\sin \pi/3)^{7/2}\right) - \left(\frac{1}{3}(\sin 0)^{3/2} - \frac{1}{7}(\sin 0)^{7/2}\right)$

$\frac{1}{3}\left(\sqrt{3}/2\right)^{3/2} - \frac{1}{7}\left(\sqrt{3}/2\right)^{7/2}$