$$\frac{1}{1-(\ )} = \sum_{n=0}^{\infty} (\ )^n$$

MATH 152
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QUIZ 9A
25 points
12.9.12.10

NAME Answers
Section__________ Wed 4/26/06

1. Find a power series representation for the functions and determine the interval of convergence. a) $f(x) = \frac{x}{2+x^2} = x\left(\frac{1/2}{1-\left(-\frac{x^2}{2}\right)}\right) =$

$$\frac{x}{2}\sum_{n=0}^{\infty}\left(-\frac{x^2}{2}\right)^n = \frac{x}{2}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2^n} = \boxed{\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2^{n+1}}}$$

$$\lim_{n\to\infty}\left|\frac{x^{2n+3}}{2^{n+2}}\cdot\frac{2^{n+1}}{x^{2n+1}}\right| = \lim_{n\to\infty}\left|\frac{x^2}{2}\right| = \left|\frac{x^2}{2}\right| < 1$$

$R = \sqrt{1/2}$

$I = [0, \sqrt{1/2}]$

$x=0 \quad \sum_{n=0}^{\infty}\frac{(-1)^n\, 0}{2^{n+1}} = 0$ so only 0's, convergent

$x=\sqrt{1/2} \quad \sum_{n=0}^{\infty}\frac{(-1)^n(\sqrt{1/2})^{2n+1}}{2^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n+3/2}} \to 0$ so convergent by AST

b) $f(x) = \ln(1-x)$

$$f'(x) = \frac{1}{1-x}(-1) = -\sum_{n=1}^{\infty} x^n$$

$$f(x) = C - \sum_{n=1}^{\infty}\frac{x^{n+1}}{n+1}$$

$$\lim_{n\to\infty}\left|\frac{x^{n+2}}{n+2}\cdot\frac{n+1}{x^{n+1}}\right| = |x| < 1$$

$R = 1$
$I = [-1, 1)$

check endpoint

$(x=-1)\ \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1} \to$ convergent by AST

$(x=1)\ \sum_{n=1}^{\infty}\frac{1^n}{n+1} \to$ divergent by comparison to Harmonic Series

c) $f(x) = \frac{1}{(4+x)^2} = (4+x)^{-2}$

$$\int f(x)\,dx = \frac{1}{-1}(4+x)^{-1} = -\frac{1}{4+x}$$

$$\frac{-1/4}{1-\left(-\frac{x}{4}\right)} = -\frac{1}{4}\sum_{n=0}^{\infty}\left(-\frac{x}{4}\right)^n = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^n}{4^{n+1}}$$

$$\frac{d}{dx}(\ ) = \boxed{\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\, n x^{n-1}}{4^{n+1}}}$$

$$\lim_{n\to\infty}\left|\frac{(n+1)x^n}{4^{n+2}}\cdot\frac{4^{n+1}}{n\,x^{n-1}}\right| = \lim_{n\to\infty}\left|\frac{x}{4}\left(\frac{n+1}{n}\right)\right|$$

$\left|\frac{x}{4}\right| < 1 \quad R = 4 \quad I = (-4, 4)$

Check endpoint

$x=-4 \quad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}\, n\,(-1)^{n-1}(4)^{n-1}}{4^{n+1}} = \sum_{n=1}^{\infty}\frac{n}{16} \to \infty$ Divergent by test for divergence

$(x=4)\ \sum_{n=1}^{\infty}\frac{(-1)^{n+1}\, n\, 4^{n-1}}{4^{n+1}} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}\, n}{16} \to \infty$ " " See above

$$f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots$$

2. Find the Taylor series for f(x) centered at the given value of a. $f(x) = \frac{1}{\sqrt{x}}$, $a = 9$ $\quad = x^{-1/2}$

$$\frac{1}{3} + \frac{-\frac{1}{2}\cdot\frac{1}{3^3}(x-a)}{1} + \frac{\frac{3}{4}\cdot\frac{1}{3^5}(x-9)^2}{2!} + \frac{-\frac{15}{8}\cdot\frac{1}{3^7}(x-9)^3}{3!}$$

$$\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{3^3} + \frac{1\cdot 3}{2^2}\cdot\frac{1}{3^5} + \frac{1\cdot 3\cdot 5}{2^3}\cdot\frac{1}{3^7} + \frac{1\cdot 3\cdot 5\cdot 7}{2^4}\cdot\frac{1}{3^9} + \cdots$$

$f'(x) = -\frac{1}{2}x^{-3/2} = -\frac{1}{2}\cdot\frac{1}{9^{3/2}} =$

$f''(x) = +\frac{3}{4}x^{-5/2} = \frac{3}{4}\cdot\frac{1}{9^{5/2}}$

$f'''(x) = -\frac{15}{8}x^{-7/2} = -\frac{15}{8}\cdot\frac{1}{9^{7/2}}$

$$\frac{1\cdot 3\cdot 5\cdots(2n-1)}{3^{2n-1}}\cdot\frac{1}{2^{n-1}}$$

$$\frac{1}{3} + \sum_{n=1}^{\infty}\frac{(-1)^n\,\bigcirc\,(x-a)^n}{n!}$$

$$\boxed{\frac{1}{3} + \sum_{n=1}^{\infty}\frac{(-1)^{n}(1\cdot 3\cdot 5\cdot 7\cdots(2n-1))(x-9)^n}{3^{2n-1}\cdot 2^{n-1}\cdot n!}}$$

3. Find the Maclaurin series of f and its radius of convergence. $f(x) = \sqrt{4+x}$ $\quad = (4+x)^{1/2} = 2$

$$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$$2 + \frac{1}{2}\cdot\frac{1}{2} + \frac{-1}{2^2}\cdot\frac{1}{2^3} + \frac{1\cdot 3}{2^3}\cdot\frac{1}{2^5} + \frac{-1\cdot 3\cdot 5}{2^4}\cdot\frac{1}{2^7} + \frac{1\cdot 3\cdot 5\cdot 7}{2^5}\left(\frac{1}{2^9}\right)$$

$f'(x) = \frac{1}{2}(4+x)^{-1/2}$ | $\frac{1}{2}\cdot\frac{1}{2}$

$f''(x) = -\frac{1}{4}(4+x)^{-3/2}$ | $-\frac{1}{4}\cdot\frac{1}{2^3}$

$f'''(x) = +\frac{3}{8}(4+x)^{-5/2}$ | $\frac{3}{8}\cdot\frac{1}{2^5}$

$f''''(x) = \frac{-1\cdot 3\cdot 5}{2^4}(4+x)^{-7/2}$ | $-\frac{15}{16}\cdot\frac{1}{2^7}$

$f^{(5)}(x) = \frac{1\cdot 3\cdot 5\cdot 7}{2^5}(4+x)^{-9/2}$ | $+\frac{1\cdot 3\cdot 5\cdot 7}{2^5}\cdot\frac{1}{2^9}$

$$\boxed{2 + \frac{1}{4} + \sum_{n=2}^{\infty}\frac{(-1)^{n+1}(1\cdot 3\cdot 5\cdots(2n-3))\,x^n}{n!\;2^{3n-1}}}$$

$$\lim_{n\to\infty}\left|\frac{1\cdot 3\cdot 5\cdots(2n-3)(2n-1)\,x^{n+1}}{(n+1)!\;2^{3n+2}}\cdot\frac{n!\;2^{3n-1}}{1\cdot 3\cdot 5\cdots(2n-3)\,x^{n}}\right| =$$

$$\lim_{n\to\infty}\left|\left(\frac{2n-1}{n+1}\right)\frac{(x)}{2^3}\right| = \left|\frac{2x}{2^3}\right| = \left|\frac{x}{4}\right| < 1 \quad \text{so } R = 4$$

2^5 2^8 2^{11} 2^{14}

$n=2$ $n=3$ $n=4$

in text

4. Evaluate the indefinite integral as an infinite series. $\int x\cos(\sqrt{x}\,)dx$

$$\boxed{\cos x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}}$$

$$\int x\sum_{n=0}^{\infty}\frac{(-1)^n(\sqrt{x})^{2n}}{(2n)!}dx = \int\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{(2n)!}dx =$$

$$\boxed{C + \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+2}}{(2n)!\,(n+2)}}$$