MATH 152
Mrs. Bonny Tighe

**QUIZ 9**
25 points
12.9.12.10

NAME Answer

Section__________ Wed 4/26/06

1. Find a power series representation for each function and determine the interval of convergence.

a) $f(x)=\dfrac{x}{4-x^2}$ $= \frac{x}{4}\left(\frac{1}{1-\frac{x^2}{4}}\right) = \frac{x}{4}\sum_{n=0}^{\infty}\left(-\frac{x^2}{4}\right)^n =$ $\boxed{\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{4^{n+1}}}$

$\lim_{x\to\infty}\left|\frac{x^{2n+1}\cdot x}{4^{n+1}\cdot 4}\cdot\frac{4^{n+1}}{x^{2n+1}}\right| = \left|\frac{x}{4}\right|<1$ $R=4$ $I=(-4,4)$

test endpoints

$(x=-4)$ $\sum_{n=0}^{\infty}\frac{(-1)^n(-1)^n(4)^{2n+1}}{4^{n+1}} = \sum_{n=0}^{\infty} 4^n \to\infty$ so divergent by the r-test, $|r|>1$

$(x=4)$ $\sum_{n=0}^{\infty}\frac{(-1)^n 4^{2n+1}}{4^{n+1}} = \sum_{n=0}^{\infty}(-1)4^n \to$ so divergent by n-test

b) $f(x)=\ln(2-x)$

$f'(x)=\frac{1}{2-x}(-1) = \frac{-1}{2}\left(\frac{1}{1-\frac{x}{2}}\right) =$

$-\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n = \int -\sum_{n=0}^{\infty}\frac{x^n}{2^{n+1}}$

$\boxed{C-\sum_{n=0}^{\infty}\frac{x^{n+1}}{(n+1)2^{n+1}}}$

$\lim_{n\to\infty}\left|\frac{x^{n+2}}{(n+2)2^{n+2}}\cdot\frac{(n+1)2^{n+1}}{x^{n+1}}\right| =$

$\lim_{n\to\infty}\left|\frac{x}{2}\frac{(n+1)}{(n+2)}\right| = \left|\frac{x}{2}\right|<1$ $R=2$ $I=[-2,2]$

$(\text{at } x=-2)$ $-\sum_{n=0}^{\infty}\frac{(-2)^{n+1}}{(n+1)2^{n+1}} = -\sum_{n=0}\frac{(-1)^{n+1}}{(n+1)2^n}$ convergent by AST

$(\text{at } x=2)$ $-\sum_{n=0}^{\infty}\frac{2^{n+1}}{(n+1)2^{n+1}} = -\sum_{n=0}^{\infty}\frac{1}{(n+1)2^n}$ = convergent comparison to $\frac{1}{n^{3/2}}$ convergent p-series

$\frac{1}{(n+1)2^n} \le \frac{1}{n^{3/2}}$, $p>1$

c) $f(x)=\dfrac{1}{(1+x)^2}$ $\int(1+x)^{-2}dx =$

$\frac{-1}{1+x} = \frac{-1}{1-(-x)} = -\sum_{n=0}^{\infty}(-x)^n$

$\frac{d}{dx}\sum_{n=0}^{\infty}(-1)^{n+1}x^n = \boxed{\sum_{n=1}^{\infty}(-1)^{n+1}nx^{n-1}}$

$\lim_{n\to\infty}\left|\frac{(n+1)x^n}{nx^{n-1}}\right| = \lim_{n\to\infty}\left|\frac{n+1}{n}\cdot x\right| = |x|<1$

$R=1$ $I=(-1,1)$

$(\text{at } x=-1)$ $\sum_{n=1}^{\infty}(-1)^{n+1}n(-1)^{n-1} =$

$\sum_{n=1} n\to\infty$ so divergent by Test for divergence

$(\text{at } x=1)$ $\sum_{n=1}^{\infty}(-1)^{n+1}n(1)^{n-1} = \sum_{n=1}^{\infty}(-1)^{n+1}n \to\infty$

so divergent

$$f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots$$

2. Find the Taylor series for f(x) centered at the given value of a. $f(x) = \frac{1}{x}$, $a = 2$

$f'(x) = -x^{-2} = -1\left(\frac{1}{2^2}\right)$

$f''(x) = +2x^{-3} = 2\left(\frac{1}{2^3}\right)$

$f'''(x) = -6x^{-4} = -6\left(\frac{1}{2^4}\right)$

$f''''(x) = 24x^{-5} = 24 = \frac{4\cdot3\cdot2\cdot1}{2^5}$

$f'''''(x) = -5\cdot4\cdot3\cdot2\cdot1\,x^{-6}$

$1 + -1 + 2 + -6 + 4! + -5!$

$$1 + \sum_{n=1}^{\infty} \frac{(-1)^n n!}{n!} \frac{(x-2)^n}{2^{n+1}} = \boxed{1 + \sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{2^{n+1}}}$$

3. Find the Maclaurin series of f and its radius of convergence. $f(x) = \sqrt{9-x} = (9-x)^{1/2} = 3$

all negative

$f'(x) = \frac{1}{2}(9-x)^{-1/2}(-1) = -\frac{1}{2}\cdot\frac{1}{3}$

$f''(x) = -\frac{1}{4}(9-x)^{-3/2}(+1) = -\frac{1}{2^2}\cdot\frac{1}{3^3}$

$f'''(x) = +\frac{3}{8}(9-x)^{-5/2}(-1) = \frac{-1\cdot3}{2^3}\cdot\frac{1}{3^5}$

$f''''(x) = \frac{-1\cdot3\cdot5}{2^4}\cdot\frac{1}{3^7}$

$$f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots$$

$$3 - \left[\frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2^2}\cdot\frac{1}{3^3} + \frac{1\cdot3}{2^3}\cdot\frac{1}{3^5} + \frac{1\cdot3\cdot5}{2^4}\cdot\frac{1}{3^7}\right]$$

$$\boxed{3 - \frac{1}{6} - \sum_{n=2}^{\infty} \frac{x^n(1\cdot3\cdot5\cdots(2n-3))}{n!\,2^n\,3^{2n-1}}}$$

$$\lim_{n\to\infty}\left|\frac{x^{n+1}(1\cdot3\cdot5\cdots(2n-3)(2n-1))}{(n+1)!\,2^{n+1}\,3^{2n+1}}\cdot\frac{n!\,2^n\,3^{2n-1}}{x^n(1\cdot3\cdot5\cdots(2n-3))}\right| \quad \lim_{n\to\infty}\left|\frac{x(2n-1)}{(n+1)(2)(9)}\right| = \left|\frac{x}{9}\right| < 1$$

$R = 9$

4. Evaluate the indefinite integral as an infinite series. $\int \sqrt{x}\, e^{-x}\,dx$

from book $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$\int x^{1/2} \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1/2}}{n!}\,dx =$$

$$\boxed{C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3/2}}{n!\,(n+3/2)}}$$