MATH 152 | **QUIZ 8A** | NAME Answers
Mrs. Bonny Tighe | 25 points
12.7, 12.8 | Section _______ Wed 4/17/06

1. Test each of the series for convergence or divergence. Show each test being used.

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln n}{n^2}$ $\boxed{\text{Comparison Test}}$

Choose $b_n = \frac{1}{n^{3/2}}$ a convergent p-series

$\frac{\ln n}{n^2} \le \frac{1}{n^{3/2}}$ so $\frac{\ln n}{n^2}$ is also Convergent

$\boxed{\text{Absolutely Convergent}}$

b) $\sum_{n=1}^{\infty} \frac{2^n}{n!(n+1)}$ $\boxed{\text{Ratio Test}}$

$\lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)!\,(n+2)} \cdot \frac{n!\,(n+1)}{2^n} \right| =$

$\lim_{n\to\infty} \left| \frac{2}{n+1} \cdot \frac{n+1}{n+2} \right| = 0 < 1$

So convergent

c) $\sum_{n=2}^{\infty} \frac{\cos \pi n}{n^2 + n}$ $\boxed{\text{Comparison Test}}$

Choose $a_n = \frac{\cos \pi n}{n^2+n}$ + $b_n = \frac{1}{n^2}$ which is convergent p-series

$\frac{\cos \pi n}{n^2+n} \le \frac{1}{n^2+1} \le \frac{1}{n^2}$ So

$\boxed{a_n \text{ is also convergent}}$

d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+2}}$ $\boxed{\text{Limit Comparison Test}}$

$a_n = \frac{1}{\sqrt{n+2}}$ $b_n = \frac{1}{\sqrt{n}}$ which is divergent p-series

$\lim_{n\to\infty} \left(\frac{\frac{1}{\sqrt{n+2}}}{\frac{1}{\sqrt{n}}} \right) = \lim_{n\to\infty} \left(\sqrt{\frac{n}{n+2}} \right) = 1 > 0$

So both are divergent But

$\boxed{\text{Conditionally convergent by AST}}$

e) $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$ $= \frac{(-1)^{n+1}\, 3^{n+1}}{2^{3n} = 8^n}$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, 9\,(3^{n-1})}{8\,(8^{n-1})} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{9}{8} \left(\frac{3}{8}\right)^{n-1}$

Geometric Series $|r| < 1$ so

Convergent.

$\boxed{\text{Absolutely Convergent}}$

f) $\sum_{n=1}^{\infty} \left(\frac{2n}{n+3} \right)^n$ $\boxed{\text{Root Test}}$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{2n}{n+3}\right)^n} =$

$\lim_{n\to\infty} \left(\frac{2n}{n+3}\right) = 2 > 1$ So

Divergent

2. Find the radius of convergence and the interval of convergence for each of the given power series.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n \ln n}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)\ln(n+1)} \cdot \frac{n \ln n}{x^n} \right| =$

$\lim_{n\to\infty} \left| x \cdot \frac{n}{n+1} \frac{\ln n}{\ln(n+1)} \right| = |x| < 1$

$R = 1 \qquad I = (-1, 1]$

$x = -1 \quad \sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{n \ln n} = \sum_{n=0}^{\infty} \frac{1}{n \ln n}$ Divergent by Integral Test

$x = 1 \quad \sum_{n=0}^{\infty} \frac{(-1)^n (1)^n}{n \ln n} \to 0$ Convergent by AST

b) $\sum_{n=0}^{\infty} \frac{(\frac{x}{3})^n}{n!} = \frac{x^n}{3^n n!}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{3^{n+1}(n+1)!} \cdot \frac{3^n n!}{x^n} \right| =$

$\lim_{n\to\infty} \left| \frac{x}{3(n+1)} \right| = 0 < 1$ always

So $R = \infty$ and

$I = (-\infty, \infty)$

c) $\sum_{n=0}^{\infty} \frac{(2x+1)^n}{n+1}$

$\lim_{n\to\infty} \left| \frac{(2x+1)^{n+1}}{n+2} \cdot \frac{n+1}{(2x+1)^n} \right| =$

$\lim_{n\to\infty} \left| 2x+1 \left(\frac{n+1}{n+2}\right) \right| = |2x+1| < 1$

$R = 1/2 \qquad I = (0, 1)$

$-1 < 2x+1 < 1$

$0 < 2x < 2$

$0 < x < 1$

$x = 0 \quad \sum_{n=0}^{\infty} \frac{(1)^n}{n+1}$ Divergent Harmonic Series

$x = 1 \quad \sum_{n=1}^{\infty} \frac{3^n}{n+1}$ Divergent

d) $\sum_{n=0}^{\infty} \frac{3^n (x-2)^n}{1 \cdot 4 \cdot 7 \cdot 10 \cdots (3n-2)}$

$\lim_{n\to\infty} \left| \frac{3^{n+1}(x-2)^{n+1}}{1 \cdot 4 \cdot 7 \cdots (3n-2)(3n+1)} \cdot \frac{1 \cdot 4 \cdot 7 (3n-2)}{3^n (x-2)^n} \right|$

$\lim_{n\to\infty} \left| \frac{3(x-2)}{3n+1} \right| = 0 < 1$ always

So $R = \infty$

$I = (-\infty, \infty)$