MATH 152
Mrs. Bonny Tighe

QUIZ 8

25 points
12.7, 12.8

NAME Answer

Section ______ Wed. 4/19/06

1. Test the series for convergence or divergence using any appropriate test. Show your work.

a) $\sum_{n=1}^{\infty}\left(\frac{3n}{1+n^2}\right)^n$ $\boxed{\text{Root Test}}$

$\lim_{n\to\infty}\sqrt[n]{\left(\frac{3n}{1+n^2}\right)^n} = \lim_{n\to\infty}\frac{3n}{1+n^2} = 0 < 1$ so

$\boxed{\text{So convergent}}$

b) $\sum_{n=1}^{\infty}\frac{n^2+n}{2^n}$ $\boxed{\text{Ratio Test}}$

$\lim_{n\to\infty}\left|\frac{(n+1)^2+n+1}{2^{n+1}}\cdot\frac{2^n}{n^2+n}\right| =$

$\lim_{n\to\infty}\left|\frac{n^2+3n+2}{n^2+n}\cdot\frac{1}{2}\right| = \frac{1}{2} < 1$

$\boxed{\text{So convergent}}$

c) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\ln n}$ $\boxed{\text{Integral Test}}$

$a_n = \frac{1}{n\ln n}$ is a positive decreasing function

$\int_1^{\infty}\frac{1}{x\ln x}\,dx = \lim_{t\to\infty}\ln(\ln x)\Big|_1^t =$

$\lim_{t\to\infty}\ln(\ln t)-\ln(\ln 1)$ undef.
∞

So divergent

d) $\sum_{n=1}^{\infty}\frac{5^n}{n!}$ $\boxed{\text{Ratio Test}}$

$\lim_{n\to\infty}\left|\frac{5^{n+1}}{(n+1)!}\cdot\frac{n!}{5^n}\right| =$

$\lim_{n\to\infty}\left|\frac{5}{n+1}\right| = 0 < 1$

$\boxed{\text{So convergent}}$

e) $\sum_{n=1}^{\infty}\frac{3^n n^2}{n!}$

$\boxed{\text{Ratio Test}}$

$\lim_{n\to\infty}\left|\frac{3^{n+1}(n+1)^2}{(n+1)!}\cdot\frac{n!}{3^n n^2}\right| =$

$\lim_{n\to\infty}\left|\frac{3}{n+1}\left(\frac{(n+1)^2}{n^2}\right)\right| = 0 < 1$

So convergent

f) $\sum_{n=1}^{\infty}\frac{n^2}{e^{2n}}$ $\boxed{\text{Ratio Test}}$

$\lim_{n\to\infty}\left|\frac{(n+1)^2}{e^{2n+2}}\cdot\frac{e^{2n}}{n^2}\right| =$

$\lim_{n\to\infty}\left|\frac{n^2+2n+1}{n^2}\cdot\frac{1}{e^2}\right| = \frac{1}{e^2} < 1$

So convergent

2. Find the radius of convergence and the interval of convergence of the series.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2+1}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^2+1} \cdot \frac{n^2+1}{x^n} \right| = \lim_{n\to\infty} \left| x \cdot \frac{n^2+1}{(n+1)^2+1} \right| =$

$|x| < 1$ $\boxed{\text{so } R=1 \quad I=[-1,1]}$

$x=-1$ $\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{1}{n^2+1}$ convergent by p-series comparison

$x=1$ $\sum_{n=0}^{\infty} \frac{(-1)^n(1)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ convergent AST

b) $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{n!}$

$\lim_{n\to\infty} \left| \frac{(2x-1)^{n+1}}{(n+1)!} \cdot \frac{n!}{(2x-1)^n} \right| =$

$\lim_{n\to\infty} \left| \frac{2x-1}{n+1} \right| = 0 < 1$ always

$\boxed{R=\infty \quad I=(-\infty,\infty)}$

c) $\sum_{n=0}^{\infty} 3n^n (x+1)^n$

$\lim_{n\to\infty} \left| \frac{3(n+1)^{n+1}(x+1)^{n+1}}{3n^n (x+1)^n} \right| =$

$\lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{n^n} \cdot (x+1) \right| = \infty \not< 1$ never

$\boxed{\text{so } R=0 \quad I=-1}$

d) $\sum_{n=0}^{\infty} \frac{3^n (x-3)^n}{n+3}$

$\lim_{n\to\infty} \left| \frac{3^{n+1}(x-3)^{n+1}}{n+4} \cdot \frac{n+3}{3^n(x-3)^n} \right| =$

$\lim_{n\to\infty} \left| 3(x-3) \cdot \frac{n+3}{n+4} \right| = |3(x-3)| < 1$

$\boxed{R=1/3 \quad I=(8/3, 10/3]}$

$-1 < 3x-9 < 1$
$8 < 3x < 10$
$8/3 < x < 10/3$

$x=8/3$ $\sum_{n=0}^{\infty} \frac{3^n(8/3-3)^n}{n+3} = \sum_{n=0}^{\infty} \frac{3^n(1/3)^n}{n+3} = \frac{1}{n+3}$ Divergent compared to Harmonic

$x=10/3$ $\sum_{n=0}^{\infty} \frac{3^n(10/3-3)^n}{n+3} =$

$\sum_{n=0}^{\infty} \frac{3^n(-1)^n(\frac{1}{3})^n}{n+3} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+3}$ Convergent by AST