MATH 152 **QUIZ 7**A NAME Answers

Mrs. Bonny Tighe 25 points

12.5, 12.6 Section _______ Wed. 4/12/06

1. Test the series for convergence or divergence. **State the test you use** and ~~show you're~~ show your work. If the series is an Alternating Series, find if it is Absolutely or Conditionally convergent.

a) $\sum_{n=1}^{\infty} \frac{3^n n}{n!}$ Ratio Test

$$\lim_{n\to\infty} \left| \frac{3^{n+1}(n+1)}{(n+1)!} \cdot \frac{n!}{3^n n} \right| =$$

$$\lim_{n\to\infty} \left| \frac{3\,\cancel{(n+1)}}{\cancel{(n+1)}(n)} \right| = 0 < 1$$

So convergent

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \ln n}{n^3}$ Ratio Test

$$\lim_{n\to\infty} \left| \frac{\ln(n+1)}{(n+1)^3} \cdot \frac{n^3}{\ln n} \right| =$$

$$\lim_{n\to\infty} \left| \frac{\ln(n+1)}{\ln(n)} \cdot \frac{n^3}{(n+1)^3} \right| =$$

$1 \cdot 1 = 1$ inconclusive

Comparison Test $\frac{\ln n}{n^3} \leq \frac{1}{n^2}$

Convergent p-series

So absolutely convergent

c) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ Ratio Test

$$\lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right| =$$

$$\lim_{n\to\infty} \left| \frac{2}{n+1} \right| = 0 < 1$$

So convergent

d) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{n^3 + 2n}$

Limit Comparison Test $a_n = \frac{1}{n^2}$

convergent p-series

$$\lim_{n\to\infty} \left(\frac{\frac{n+1}{n^3+2n}}{\frac{1}{n^2}} \right) =$$

$$\lim_{n\to\infty} \left(\frac{n+1}{n^3+2n} \cdot \frac{n^2}{1} \right) =$$

$$\lim_{n\to\infty} \left(\frac{n^3+n}{n^3+2n} \right) = 1 > 0 \text{ finite}$$

So both are convergent

So absolutely convergent.

e) $\sum_{n=1}^{\infty}\frac{n!}{e^n n}$ $\boxed{\text{Ratio Test}}$

$$\lim_{n\to\infty}\left|\frac{(n+1)!}{e^{n+1}(n+1)}\cdot\frac{e^n n}{n!}\right|$$

$$\lim_{n\to\infty}\left|\frac{\cancel{n+1}\;(n)}{e\,(\cancel{n+1})}\right|$$

$$\lim_{n\to\infty}\left|\frac{n}{e}\right| = \infty > 1$$

So divergent

f) $\sum_{n=1}^{\infty}\left(\frac{3n}{1+8n}\right)^n$ $\boxed{\text{Root Test}}$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{3n}{1+8n}\right)^n} =$$

$$\lim_{n\to\infty}\left(\frac{3n}{1+8n}\right) = 3/8 \neq 0$$

$\boxed{\text{So divergent}}$

g) $\sum_{n=1}^{\infty}\frac{(-2)^n}{5^{n-1}}$ $\frac{(-1)^n 2^n}{5^{n-1}}$

$\boxed{\text{Geometric Series}}$

$$\sum_{n=1}^{\infty} 2\left(\frac{2}{5}\right)^{n-1}$$

$|r|<1$

$\boxed{\text{So absolutely Convergent}}$

$\boxed{\text{Alternating Series Test}}$

$$\lim_{n\to\infty}\frac{1}{(n+1)!} = 0 \checkmark$$

h) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(n+1)!}$ So conditionally convergent

$\boxed{\text{Ratio Test}}$

$$\lim_{n\to\infty}\left|\frac{1}{(n+1)!(n+2)}\cdot\frac{(n+1)!}{1}\right| =$$

$$\lim_{n\to\infty}\left|\frac{1}{n+2}\right| = 0 < 1$$

So

$\boxed{\text{absolutely convergent}}$