MATH 152 **QUIZ 7** NAME Answers

Mrs. Bonny Tighe 25 points

12.5, 12.6 Section _______ Wed. 4/12/06

1. Test the series for convergence or divergence. **State the test you use** and show ~~you're~~ your work. If the series is an Alternating Series, find if it is Absolutely or Conditionally convergent.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n^3+2n}$

Comparison Test

Choose $a_n = \frac{1}{n^2}$, convergent p-series

$\frac{n-1}{n^3+2n} \le \frac{1}{n^2}$ for all n

So $\frac{n-1}{n^3+2n}$ is also convergent

So absolutely convergent

b) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n5^{n-1}}$ $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n 5^{n-1}}$

Ratio Test

$\lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)5^n} \cdot \frac{n5^{n-1}}{2^n} \right| =$

$\lim_{n\to\infty} \left| \frac{2}{5}\left(\frac{n}{n+1}\right) \right| = 2/5 < 1$

So absolutely convergent

c) $\sum_{n=1}^{\infty} \frac{10^n}{n!}$

Ratio Test

$\lim_{n\to\infty} \left| \frac{10^{n+1}}{(n+1)!} \cdot \frac{n!}{10^n} \right| =$

$\lim_{n\to\infty} \left| \frac{10}{n+1} \right| = 0 < 1$

So convergent

d) $\sum_{n=1}^{\infty} \frac{2^n n}{n!}$

Ratio Test

$\lim_{n\to\infty} \left| \frac{2^{n+1}(n+1)}{(n+1)!} \cdot \frac{n!}{2^n n} \right|$

$\lim_{n\to\infty} \left| \frac{2\cancel{(n+1)}}{\cancel{n+1}(n)} \right| = \lim_{n\to\infty} \left| \frac{2}{n} \right| = 0$

$0 < 1$

So Convergent

Ratio Test

e) $\sum_{n=1}^{\infty} \frac{n!}{e^n}$

$\lim_{n\to\infty} \left| \frac{(n+1)!}{e^{n+1}} \cdot \frac{e^n}{n!} \right| =$

$\lim_{n\to\infty} \left| \frac{n+1}{e} \right| = \infty > 1$

So divergent

Root Test

f) $\sum_{n=1}^{\infty} \left(\frac{3}{1+8n} \right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{3}{1+8n}\right)^n} =$

$\lim_{n\to\infty} \left(\frac{3}{1+8n} \right) = 0$

So convergent

Ratio Test

g) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \ln n}{n^2}$

$\lim_{n\to\infty} \left| \frac{\ln(n+1)}{(n+1)^2} \cdot \frac{n^2}{\ln n} \right|$

$\lim_{n\to\infty} \left| \frac{\ln(n+1)}{\ln n} \cdot \frac{n^2}{(n+1)^2} \right| = 1$

inconclusive

Integral Test

$\lim_{t\to\infty} \int_1^t \underset{u}{\ln x} \ \underset{dv}{\frac{1}{x^2}\,dx}$

$du = \frac{1}{x} \quad v = -\frac{1}{x}$

$\lim_{t\to\infty} -\frac{\ln x}{x} - \int_1^t -\frac{1}{x} \cdot \frac{1}{x}\,dx \quad \lim_{t\to\infty} \left(-\frac{\ln x}{x} - \frac{1}{x} \right) \Big|_1^t$

$\lim_{t\to\infty} \left(-\frac{\ln t}{t} - \frac{1}{t} \right) - \left(-\frac{\ln 1}{1} - \frac{1}{1} \right) = (0 - 0 + 1) = 1$

So absolutely convergent

h) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+1)!}$

Ratio Test

$\lim_{n\to\infty} \left| \frac{1}{(n+2)!} \cdot \frac{(n+1)!}{1} \right| =$

$\lim_{n\to\infty} \left| \frac{1}{n+2} \right| = 0 < 1$

So absolutely convergent