MATH 152 **QUIZ 6A** NAME Answers

Mrs. Bonny Tighe 25 points

9.1,9.2,12.1,12.2 Section _____ Wed. 3/29/06

1. Find the area of the surface obtained by rotation the curve about the y-axis.

$x = y^3, \ 0 \le y \le 2$

$$L = 2\pi\int_0^2 x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy = 2\pi\int_0^2 y^3\sqrt{1+(3y^2)^2}\,dy =$$

$$\frac{2\pi}{36}\int (1+9y^4)^{1/2}\,36y^3\,dy = \frac{\pi}{18}\cdot\frac{1}{3/2}(1+9y^4)^{3/2}\Big|_0^2 =$$

$$\frac{\pi}{27}(1+9y^4)^{3/2}\Big|_0^2 = \frac{\pi}{27}\left[(1+9(16))^{3/2} - (1+9(0))^{3/2}\right] = \frac{\pi}{27}\left[(145)^{3/2} - 1\right]$$

2. Determine whether the given sequence is increasing, decreasing or not monotonic. Is the sequence bounded?

a) $a_n = \dfrac{2}{n+1}$

$\frac{2}{2}, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}$

monotonic, increasing and bounded

b) $a_n = 3 + (-1)^{n+1}$

not monotonic

$3+1, 3-1, 3+1, 3-1$

$4, 2, 4, 2, 4, 2$

yes bounded (up by 4, down by 2)

3. Find the length of the curve: $y = 3x^{3/2} - 1, \ 0 \le x \le 1$

$$\frac{dy}{dx} = 3\cdot\frac{3}{2}x^{1/2} = \frac{9}{4}\sqrt{x}$$

$$\int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 \sqrt{1+\frac{81}{16}x}\,dx = \frac{16}{81}\int_0^1 \sqrt{1+\frac{81}{16}x}\,\frac{81}{16}\,dx$$

$$\frac{16}{81}\cdot\frac{1}{3/2}\left(1+\frac{81}{16}x\right)^{3/2}\Big|_0^1 = \frac{16}{81}\cdot\frac{2}{3}\left(1+\frac{81}{16}x\right)^{3/2}\Big|_0^1 =$$

$$\frac{32}{243}\left(\left(1+\frac{81}{16}\right) - (1+0)\right) = \frac{32}{243}\left(\frac{97}{16} - 1\right) = \frac{32}{243}\left(\frac{81}{16}\right) = \frac{2}{3}$$

4. Determine whether the sequence converges or diverges. If it converges, find the limit.

a) $a_n = \dfrac{(n+2)}{2(n+1)^2}$

$\lim_{n\to\infty} \left(\dfrac{n+2}{2(n+1)^2}\right) = 0$

So convergent

b) $a_n = \sin(n\pi)$

$\lim_{n\to\infty} (\sin n\pi) = 0$

So convergent

c) $a_n = \dfrac{2^{n+1}}{7^n}$

$\lim_{n\to\infty} \left(\dfrac{2^{n+1}}{7^n}\right) = 0$

So Convergent

d) $a_n = \ln(n+2) - \ln(n-1)$

$\lim_{n\to\infty} \ln\left(\dfrac{n+2}{n-1}\right) = \ln(1) = 0$

So convergent

5. Determine whether the series is divergent or convergent. If it is convergent, find its sum.

a) $\sum_{n=1}^{\infty} 5^{-n} 7^{n-1} = \dfrac{7^{n-1}}{5^n}$

$\lim_{n\to\infty} \left(\dfrac{7^{n-1}}{5^n}\right) = \infty \neq 0$

So divergent by The Test for Divergence

b) $\sum_{n=1}^{\infty} \dfrac{1}{n^2+n}$ $\quad \sum_{n=1}^{\infty} \left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$ Telescoping + convergent

Sum $= \dfrac{1}{1} + \cancel{\dfrac{1}{2}} + \cancel{\dfrac{1}{3}} + \cancel{\dfrac{1}{4}} + \cdots$
$\quad - \cancel{\dfrac{1}{2}} - \cancel{\dfrac{1}{3}} - \cancel{\dfrac{1}{4}} - \dfrac{1}{5}$

$=1$

c) $\sum_{n=1}^{\infty} \dfrac{n-1}{n+1}$

$\lim_{n\to\infty} \left(\dfrac{n-1}{n+1}\right) = 1 \neq 0$

So divergent by The Test for Divergence

d) $1 + \dfrac{1}{\sqrt{2}} + \dfrac{1}{2} + \dfrac{1}{2\sqrt{2}} + \dfrac{1}{4} + \ldots = \sum_{n=1}^{\infty} \left(\dfrac{1}{\sqrt{2}}\right)^{n-1}$

geometric Series, $|r|<1$

So convergent

Sum $= \dfrac{a}{1-r}$

$\dfrac{1}{1-1/\sqrt{2}} = \dfrac{\sqrt{2}}{\sqrt{2}-1}$

6. Express the number $0.\overline{54}$ as a ratio of integers.

$\dfrac{54}{100} + \dfrac{54}{10000} + \dfrac{54}{1000000} \cdots = \sum_{n=1}^{\infty} \dfrac{54}{(100)^n} = \sum_{n=1}^{\infty} \dfrac{54}{100}\left(\dfrac{1}{100}\right)^{n-1}$

geometric Series with $|r|<1$

Sum $= \dfrac{\frac{54}{100}}{1 - 1/100} = \dfrac{54}{99} = \dfrac{6}{11}$