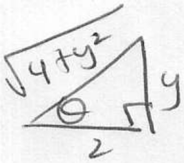
MATH 152
Mrs. Bonny Tighe

QUIZ 6

25 points
9.1,9.2,12.1,12.2

NAME Answers

Section _____ Wed. 3/29/06



$y = 2\tan\theta$
$dy = 2\sec^2\theta\, d\theta$

1. Find the length of the curve: $4x = y^2$, $0 \le y \le 2$

$x = \frac{1}{4}y^2$

$\frac{dx}{dy} = \frac{1}{2}y$

$$L = \int_c^d \sqrt{1+(dx/dy)^2}\,dy = \int_0^2 \sqrt{1+(\tfrac{1}{2}y)^2}\,dy = \int_0^2 \frac{\sqrt{4+y^2}}{2}\,dy$$

$$\int_0^2 \frac{\sqrt{4+4\tan^2\theta}\,(2\sec^2\theta)\,d\theta}{2} = 2\int \sec^3\theta\, d\theta = 2\int \underbrace{\sec^2\theta}_{du}\,\underbrace{\sec\theta}_{v} =$$

$$\sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta = \sec\theta\tan\theta - \int \sec^3\theta + \int \sec\theta\, d\theta$$

$$\cancel{3}\int \sec^3\theta = \frac{1}{3}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right)\Big|_0^2 = \frac{1}{3}\left[\frac{\sqrt{4+y^2}}{2}\cdot\frac{y}{2} + \ln\left|\frac{\sqrt{4+y^2}}{2} + \frac{y}{2}\right|\right]_0^2$$

$$= \sqrt{2} + \ln(1+\sqrt{2})$$

2. Find the area of the surface obtained by rotation the curve about the y-axis.
$x = y^3$, $0 \le y \le 2$

$$2\pi\int_c^d x\sqrt{1+(dy/dx)^2}\,dy$$

$$2\pi\int_0^2 y^3\sqrt{1+(3y^2)^2}\,dy = \frac{2\pi}{36}\int_0^2 36y^3(1+9y^4)^{1/2}\,dy$$

$$\frac{\pi}{18}\cdot\frac{1}{3/2}(1+9y^4)^{3/2}\Big|_0^2 = \frac{\pi}{27}(1+9y^4)^{3/2}\Big|_0^2 =$$

$$\frac{\pi}{27}\left((1+9(16))^{3/2} - (1+0)^{3/2}\right) =$$

$$\pi/27\left((145)^{3/2} - 1\right)$$

3. Express the number $0.\overline{15}$ as a ratio of integers.

$$\frac{15}{100} + \frac{15}{1000} + \frac{15}{1000000}\cdots = \sum_{n=1}^{\infty}\frac{15}{10^{2n}} = \sum_{n=1}^{\infty}\frac{15}{100}\left(\frac{1}{10^2}\right)^{n-1} = ar^{n-1} = \frac{a}{1-r}$$

$$\frac{\frac{15}{100}}{1-\frac{1}{100}} = \frac{15}{99} = \frac{5}{33}$$

3. Determine whether the sequence converges or diverges. If it converges, find the limit.

a) $a_n = \dfrac{n(n+2)}{3(n-1)^2}$

$\lim_{n\to\infty} \dfrac{n^2+2n}{3n^2-6n+3} = \dfrac{1}{3}$

So convergent

b) $a_n = \cos(n\pi/2) =$

$\lim_{n\to\infty} \cos(n\pi/2) = 0$, so convergent

c) $a_n = \dfrac{3^{n+2}}{5^n} = 9\left(\dfrac{3}{5}\right)^n = \dfrac{27}{5}\left(\dfrac{3}{5}\right)^{n-1}$

Converges

$\lim_{n\to\infty} \left(\dfrac{3^{n+2}}{5^n}\right) = 0$ so convergent

d) $a_n = \ln(n+2) - \ln(n-1)$

$\lim_{n\to\infty} \left[\ln\left(\dfrac{n+2}{n-1}\right)\right] = \ln(1) = 0$

So convergent

5. Determine whether the given sequence is increasing, decreasing or not monotonic. Is the sequence bounded?

a) $a_n = \dfrac{3}{n+1}$

$\frac{3}{2}$ $\frac{3}{3}$ $\frac{3}{4}$ $\frac{3}{5}$ $\frac{3}{6}$ $\frac{3}{7}$

monotonic, decreasing + bounded

b) $a_n = 2N + (-1)^n$

not monotonic

$2-1, 4+1, 6-1, 8+1, 10-1$

$1, 5, 5, 9, 9$

not bounded

6. Determine whether the series is divergent or convergent. If it is convergent, find its sum.

a) $\sum_{n=1}^{\infty} 3^{-n}7^{n-1} = \dfrac{7^{n-1}}{3^n} = \dfrac{1}{3}\left(\dfrac{7}{3}\right)^{n-1}$

Divergent by test for divergence

$\lim_{n\to\infty} \frac{1}{3}\left(\frac{7}{3}\right)^{n-1} = \infty$

$\infty \neq 0$

b) $\sum_{n=1}^{\infty} \dfrac{1}{n^2+n} = \sum_{n=1}^{\infty} \left(\dfrac{-1}{n+1} + \dfrac{1}{n}\right)$ telescoping

Convergent

Sum $= -\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} - \cancel{\frac{1}{6}} \ldots$

$\frac{1}{1} + \cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}}$

$= 1$

c) $\sum_{n=1}^{\infty} \dfrac{2n-1}{n+1}$

Divergent by test for divergence

$\lim_{n\to\infty} \left(\dfrac{2n-1}{n+1}\right) = 2 \neq 0$

So divergent

d) $1 + \dfrac{1}{\sqrt{3}} + \dfrac{1}{3} + \dfrac{1}{3\sqrt{3}} + \dfrac{1}{9} + \ldots$

$\sum_{n=1}^{\infty} \left(\dfrac{1}{\sqrt{3}}\right)^{n-1}$

geometric series $|r| < 1$

So convergent

Sum $= \dfrac{a}{1-r} = \dfrac{1}{1 - 1/\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{3}-1}$