MATH 152
Mrs. Bonny Tighe

QUIZ 5A

25 points
8.6, 8.7, 8.8

NAME Answers

Section________ Wed. 3/15/06

1. Use (a) the Midpoint Rule (b) the Trapezoidal Rule and (c) Simpson's Rule to approximate the given integral with the specified value of n. And check how close your approximations are to the actual area under the curve.

$\int_0^2 (x+x^2)dx, \quad n=6$

$\frac{1}{6} \quad \frac{3}{6} \quad \frac{5}{6} \quad \frac{7}{6} \quad \frac{9}{6} \quad \frac{11}{6}$ ← Midpoints

$0 \quad \frac{1}{3} \quad \frac{2}{3} \quad 1 \quad \frac{4}{3} \quad \frac{5}{3} \quad 2$

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6$

$$\Delta x = \frac{2-0}{6} = \frac{1}{3}$$

$$M_6 = \Delta x\left[f(\tfrac{1}{6})+f(\tfrac{3}{6})+f(\tfrac{5}{6})+f(\tfrac{7}{6})+f(\tfrac{9}{6})+f(\tfrac{11}{6})\right]$$

$$= \frac{1}{3}\left[\frac{1}{6}+\frac{1}{36}+\frac{3}{6}+\frac{9}{36}+\frac{5}{6}+\frac{25}{36}+\frac{7}{6}+\frac{49}{36}+\frac{9}{6}+\frac{81}{36}+\frac{11}{6}+\frac{121}{36}\right] =$$

$$= \frac{1}{3}\left[\frac{36}{6}+\frac{286}{36}\right] = 2+\frac{286}{108} = \frac{502}{108} = 4\,\tfrac{70}{108} = 4\,\tfrac{35}{54}$$

$$T_6 = \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+2f(x_4)+2f(x_5)+f(x_6)\right]$$

$$= \frac{1}{6}\left[0+2(\tfrac{1}{3}+\tfrac{1}{9})+2(\tfrac{2}{3}+\tfrac{4}{9})+2(1+1)+2(\tfrac{4}{3}+\tfrac{16}{9})+2(\tfrac{5}{3}+\tfrac{25}{9})+(2+4)\right]$$

$$= \frac{1}{6}\left[\frac{2}{3}+\frac{2}{9}+\frac{4}{3}+\frac{8}{9}+4+\frac{8}{3}+\frac{32}{9}+\frac{10}{3}+\frac{50}{9}+6\right]$$

$$\frac{1}{6}\left[10+\frac{24}{3}+\frac{92}{9}\right] = \frac{1}{6}\left[18+\frac{92}{9}\right] = 3+\frac{92}{54} = 4\,\tfrac{38}{54}$$

$$S_6 = \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+4f(x_5)+f(x_6)\right]$$

$$= \frac{1}{9}\left[0+4(\tfrac{1}{3}+\tfrac{1}{9})+2(\tfrac{2}{3}+\tfrac{4}{9})+4(1+1)+2(\tfrac{4}{3}+\tfrac{16}{9})+4(\tfrac{5}{3}+\tfrac{25}{9})+(2+4)\right]$$

$$= \frac{1}{9}\left[\frac{4}{3}+\frac{4}{9}+\frac{4}{3}+\frac{8}{9}+8+\frac{8}{3}+\frac{32}{9}+\frac{20}{3}+\frac{100}{9}+6\right] = \frac{1}{9}\left[14+\frac{36}{3}+\frac{144}{9}\right] =$$

$$\frac{1}{9}\left[14+12+16\right] = \frac{42}{9} = 4\,\tfrac{2}{3}$$

Check: $\int_0^2 (x+x^2)dx = \frac{1}{2}x^2+\frac{1}{3}x^3\Big|_0^2 = \left(\frac{1}{2}(4)+\frac{8}{3}\right)-(0) = 2+2\tfrac{2}{3} = 4\tfrac{2}{3}$

2. Determine whether each integral is convergent or divergent. Evaluate those that are convergent..

a) $\int_1^{\infty} \frac{2x}{(x^2+1)^2}\,dx$ $\quad (x^2+1)^{-2}\,2x\,dx$

$\lim_{t\to\infty} \int_1^t u^{-2}\,du = \lim_{t\to\infty} \frac{1}{-1}(x^2+1)^{-1} \Big|_1^t$

$\lim_{t\to\infty} \left(\frac{-1}{(t^2+1)} - \frac{-1}{(1+1)}\right) = 0 + \frac{1}{2}$

$\frac{1}{2}$

So convergent

b) $\int_{-1}^{1} \frac{2}{\sqrt{3x+3}}\,dx$

$\lim_{t\to -1} \frac{2}{3}\int_t^1 (3x+3)^{-1/2}\,3\,dx$

$\lim_{t\to -1} \left[\frac{2}{3}\,\frac{1}{1/2}(3x+3)^{1/2}\right]_t^1 =$

$\lim_{t\to -1} \left(\frac{4}{3}\sqrt{6} - \frac{4}{3}\sqrt{t}\right]$

undefined

So divergent

c) $\int_0^{\infty} \cos^2 x\,dx$

$\lim_{t\to\infty} \int_0^t \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx$

$\lim_{t\to\infty} \frac{1}{2}x + \frac{1}{4}\sin 2x \Big|_0^t$

$\lim_{t\to\infty} \left(\frac{1}{2}t + \frac{1}{4}\sin t\right) - (0)$

∞

divergent

d) $\int_0^1 x^2 \ln x\,dx$

parts

$\lim_{t\to 0} \int_t^1 x^2 \ln x$

$dv \quad u \quad du = \frac{1}{x}$

$v = \frac{1}{3}x^3$

$\lim_{t\to 0} \ln x\left(\frac{1}{3}x^3\right) - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$

$\lim_{t\to 0} \frac{1}{3}x^3(\ln x) - \frac{1}{3}\cdot\frac{1}{3}x^3 \Big|_t^1$

$\lim_{t\to 0} \left(\frac{1}{3}\ln(1) - \frac{1}{9}(1)\right) - \left(\frac{1}{3}t^3\ln t - \frac{1}{9}t^3\right)$

$\lim_{t\to 0} \left(-\frac{1}{9} - \frac{1}{3}t^3\ln t - \frac{1}{9}t^3\right)$

undefined

So divergent