MATH 152
Mrs. Bonny Tighe

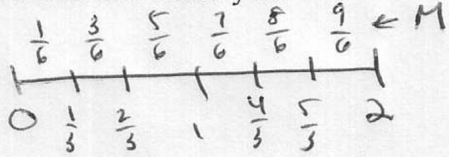
QUIZ ~~8~~ 5
25 points
8.6, 8.7, 8.8

NAME Answers

Section________ Wed. 3/15/06

1. Use (a) the Midpoint Rule (b) the Trapezoidal Rule and (c) Simpson's Rule to approximate the given integral with the specified value of n. And check how close your approximations are to the actual area under the curve.

$\int_0^2 (x^2+1)dx, \quad n = 6$ $\qquad \Delta x = \frac{2-0}{6} = \frac{1}{3}$



$M_6 = \frac{1}{3}\left[f(\tfrac{1}{6}) + f(\tfrac{3}{6}) + f(\tfrac{5}{6}) + f(\tfrac{7}{6}) + f(\tfrac{8}{6}) + f(\tfrac{9}{6})\right]$

$= \frac{1}{3}\left[\frac{1}{36}+1 + \frac{9}{36}+1 + \frac{25}{36}+1 + \frac{49}{36}+1 + \frac{64}{36}+1 + \frac{81}{36}+1\right] = \frac{1}{3}\left[6 + \frac{229}{36}\right] = 2 + \frac{229}{108}$

$\frac{445}{108}$

$4\,\frac{13}{108}$

$T_6 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots f(x_n)\right] =$

$= \frac{1}{6}\left[f(0) + 2f(\tfrac{1}{3}) + 2f(\tfrac{2}{3}) + 2f(1) + 2f(\tfrac{4}{3}) + 2f(\tfrac{5}{3}) + f(2)\right] =$

$= \frac{1}{6}\left[0+1 + 2(\tfrac{1}{9}+1) + 2(\tfrac{4}{9}+1) + 2(1+1) + 2(\tfrac{16}{9}+1) + 2(\tfrac{25}{9}+1) + (5)\right] =$

$= \frac{1}{6}\left[1 + \frac{2}{9} + 2 + \frac{8}{9} + 2 + 4 + \frac{32}{9} + 2 + \frac{50}{9} + 2 + 5\right] = \frac{1}{6}\left[18 + \frac{92}{9}\right] = 3 + \frac{92}{54} = 4\,\frac{38}{54}$

$S_6 = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots 4f(x_{n-1}) + f(x_n)\right] =$

$\frac{1}{9}\left[1 + 4(\tfrac{1}{9}+1) + 2(\tfrac{4}{9}+1) + 4(1+1) + 2(\tfrac{16}{9}+1) + 4(\tfrac{25}{9}+1) + 5\right]$

$\frac{1}{9}\left[1 + \frac{4}{9} + 4 + \frac{8}{9} + 2 + 8 + \frac{32}{9} + 2 + \frac{100}{9} + 4 + 5\right] = \frac{1}{9}\left[26 + \frac{144}{9}\right] = \frac{26}{9} + \frac{16}{9} = \frac{42}{9}$

$4\,\frac{2}{3}$

$\int_0^2 (x^2+1)dx = \frac{1}{3}x^3 + x \Big|_0^2 =$

$(\frac{1}{3}(8)+2) - (0) = \frac{8}{3} + 2 = 4\frac{1}{3}$

check

2. Determine whether each integral is convergent or divergent. Evaluate those that are convergent..

a) $\int_1^{\infty} \frac{2}{(x\ +1)^2}\, dx$

$\lim_{t\to\infty} \int_1^t 2(x+1)^{-2}\,dx$

$\lim_{t\to\infty} \frac{2}{-1}(x+1)^{-1}\Big|_1^t =$

$\lim_{t\to\infty} \frac{-2}{x+1}\Big|_1^t$

$\lim_{t\to 0} \left(\frac{-2}{t+1} - \left(\frac{-2}{2}\right)\right) = +1$

So Convergent

b) $\int_{-3}^{1} \frac{2}{\sqrt{x+3}}\,dx$

$\lim_{t\to -3} \int_t^1 2(x+3)^{-1/2}\,dx =$

$\lim_{t\to -3} 2\cdot\frac{1}{1/2}(x+3)^{1/2}\Big|_t^1 =$

$\lim_{t\to -3} \left(4\sqrt{1+3} - 4\sqrt{t+3}\right) =$

$4(2) - 4\sqrt{0} = 8$

So convergent

c) $\int_0^1 \frac{e^x}{e^x - 1}\,dx$

$\lim_{t\to 0} \int_t^1 \frac{1}{e^x-1} e^x\,dx$

$\lim_{t\to 0} \ln|e^x-1|\Big|_t^1$

$\lim_{t\to 0} \ln|e^1-1| - \ln|e^t-1|$

$\lim_{t\to 0} \ln|e-1| - \ln|e^t-1|$ undefined

So divergent

d) $\int_2^{\infty} \frac{1}{x\ln x}\,dx$

$\lim_{t\to\infty} \int_2^t \frac{1}{u}\,du =$

$\lim_{t\to\infty} \ln|\ln x|\Big|_2^t =$

$\lim_{t\to\infty} \left(\ln|\ln t| - \ln|\ln 2|\right)$

$\lim_{t\to\infty} = \infty$

Divergent