

MATH 152
Mrs. Bonny Tighe

QUIZ 4
25 points
8.4, 8.5

NAME Answers
Section _____ Wed 3/8/06

1. Evaluate the integrals.

a)

$$\int_0^5 \frac{3x-1}{x+2} dx$$

$$\begin{array}{r} 3 \\ x+2 \overline{) 3x-1} \\ \underline{-(3x+6)} \\ -7 \end{array}$$

$$\int_0^5 \left(3 - \frac{7}{x+2} \right) dx = 3x - 7 \ln|x+2| \Big|_0^5 =$$

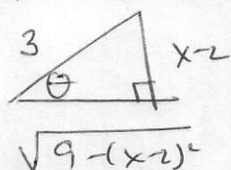
$$(15 - 7 \ln(7)) - (0 - 7 \ln(2)) =$$

$$15 - 7 \ln 7 + 7 \ln 2$$

$$b) \int \sqrt{5+4x-x^2} dx = \int \sqrt{9-(x-2)^2} dx$$

$$\begin{array}{l} -(x^2 - 4x + 4) + 5 + 4 \\ -(x-2)^2 + 9 \end{array}$$

$$\left. \begin{array}{l} (x-2) = 3 \sin \theta \\ dx = 3 \cos \theta d\theta \end{array} \right\}$$



$$\int \sqrt{9-9\sin^2 \theta} \cdot 3 \cos \theta d\theta$$

$$\int 3 \cos \theta \cdot 3 \cos \theta d\theta$$

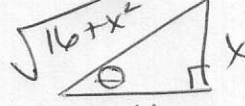
$$9 \int \cos^2 \theta d\theta = 9 \int \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) d\theta$$

$$\int \left(\frac{9}{2} + \frac{9}{2} \cos 2\theta \right) d\theta = \frac{9}{2} \theta + \frac{9}{4} \sin 2\theta + C$$

$$\frac{9}{2} \theta + \frac{9}{2} \sin \theta \cos \theta + C = \frac{9}{2} \sin^{-1} \left(\frac{x-2}{3} \right) + \frac{9}{2} \left(\frac{x-2}{3} \cdot \frac{\sqrt{9-(x-2)^2}}{3} \right) + C$$

$$\frac{9}{2} \sin^{-1} \left(\frac{x-2}{3} \right) + \frac{(x-2)\sqrt{9-(x-2)^2}}{2} + C$$

$$c) \int \frac{\sqrt{16+x^2}}{x} dx$$



$$\begin{cases} x = 4 \tan \theta \\ dx = 4 \sec^2 \theta \end{cases}$$

$$\begin{aligned} \int \frac{\sqrt{16+16\tan^2\theta}}{4\tan\theta} \cdot 4\sec^2\theta &= \int \frac{4\sec^3\theta}{\tan\theta} d\theta = 4 \int \frac{\sec^2\theta}{\tan\theta} \frac{\csc\theta d\theta}{u} \quad (\text{parts}) \\ &= \csc\theta \tan\theta - \int \tan\theta (-\csc\theta \cot\theta) d\theta = \csc\theta \tan\theta + \int \csc\theta d\theta \\ &= \csc\theta \tan\theta + \ln|\csc\theta - \cot\theta| + C \\ &= \left(\frac{\sqrt{16+x^2}}{x} \right) \left(\frac{x}{4} \right) + \ln \left| \frac{\sqrt{16+x^2}}{x} - \frac{4}{x} \right| + C \end{aligned}$$

$$\frac{\sqrt{16+x^2}}{4} + \ln \left| \frac{\sqrt{16+x^2} - 4}{x} \right| + C$$

$$d) \int \frac{2x-1}{x^2(x-1)} dx \quad \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{2x-1}{x^2(x-1)}$$

$$A(x)(x-1) + B(x-1) + Cx^2 = 2x-1$$

$$\begin{aligned} (x=0) \quad B(-1) &= -1 \quad \boxed{B=1} & A+B &= 0 \quad x^2 \\ (x=1) \quad \boxed{C} &= 1 & A &= -1 \end{aligned}$$

$$\int \left(\frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x-1} \right) dx =$$

$$- \ln|x| + \frac{1}{-1} x^{-1} + \ln|x-1| + C$$

$$- \ln|x| - \frac{1}{x} + \ln|x-1| + C$$

e) $\int \frac{5x^2 - 6}{2x^2 - 5x - 3} dx$
 $(2x+1)(x-3)$

$$\frac{A}{2x+1} + \frac{B}{x-3} = \frac{25x + 17/2}{2x^2 - 5x - 3}$$

$$2A(x-3) + 2B(2x+1) = 25x + 17$$

$(x=3) \quad 2B(7) = 75 + 17 = 92$
 $B = 92/14 = 46/7$

$(x=-1/2) \quad 2A(-7/2) = -25/2 + 17$
 $-7A = 5/2$
 $A = -5/14$

$$2x^2 - 5x - 3 \overline{) 5x^2 - 6} \begin{array}{r} 5/2 \\ \underline{5x^2 - 25/2x - 17/2} \\ 25/2x + 17/2 \end{array}$$

$$\int \frac{5}{2} dx - \frac{5}{14} \int \frac{1}{2x+1} dx + \frac{46}{7} \int \frac{1}{x-3} dx$$

$$\frac{5}{2}x - \frac{5}{14} \cdot \frac{1}{2} \ln|2x+1| + \frac{46}{7} \ln|x-3| + C$$

$$\frac{5}{2}x - \frac{5}{28} \ln|2x+1| + \frac{46}{7} \ln|x-3| + C$$

f) $\int \frac{e^x}{e^{2x} - e^x - 6} dx$

$$\begin{cases} u = e^x \\ \ln u = x \\ \frac{1}{u} du = dx \end{cases}$$

$$= \int \frac{u}{u^2 - u - 6} \left(\frac{1}{u} du \right)$$

$$\int \frac{1}{(u-3)(u+2)} du$$

$$= \frac{1}{5} \int \frac{1}{u-3} du - \frac{1}{5} \int \frac{1}{u+2} du$$

$$\frac{1}{5} \ln|u-3| - \frac{1}{5} \ln|u+2| + C$$

$$\frac{1}{5} \ln|e^x - 3| - \frac{1}{5} \ln|e^x + 2| + C$$

$$\frac{A}{u-3} + \frac{B}{u+2} = \frac{1}{(u-3)(u+2)}$$

$$A(u+2) + B(u-3) = 1$$

$(u=3) \quad 5A = 1, A = 1/5$
 $(u=-2) \quad -5B = 1, B = -1/5$