MATH 152
Mrs. Bonny Tighe

QUIZ 3A
7.7, 8.1
25 points

NAME Answers

Section ________ Wed. 2/22/06

1. Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

a) $\lim\limits_{x\to 2} \dfrac{3x}{x-2} =$ DNE

$\frac{6}{0}$

b) $\lim\limits_{x\to\infty} \dfrac{e^x}{x^3} =$ ∞

$\frac{\infty}{\infty}$ L'H $= \lim\limits_{x\to\infty} \frac{e^x}{3x^2} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim\limits_{x\to\infty} \left(\frac{e^x}{6x}\right) = \frac{\infty}{\infty} \overset{L'H}{=} \lim\limits_{x\to\infty} \frac{e^x}{6} = \infty$

c) $\lim\limits_{x\to 0} \dfrac{x}{\sin^{-1} x} =$ 1

$\frac{0}{0} \overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{1}{\frac{1}{\sqrt{1-x^2}}}$

$\lim\limits_{x\to 0} \sqrt{1-x^2} = 1$

d) $\lim\limits_{x\to\infty} \dfrac{(\ln x)^2}{x} =$ 0

$\left(\lim\limits_{x\to\infty} \frac{\ln x}{\sqrt{x}}\right)^2 = \frac{\infty}{\infty}$ L'H

$\left(\lim\limits_{x\to\infty} \left(\dfrac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}\right)\right)^2 = \left(\lim\limits_{x\to\infty} \left(\frac{1}{x} \cdot \frac{2\sqrt{x}}{*}\right)\right)^2$

e) $\lim\limits_{x\to 0^+} \sqrt{x}\ln x =$ 0

$\lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{\sqrt{x}}} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim\limits_{x\to 0^+} \left(\dfrac{\frac{1}{x}}{-\frac{1}{2x^{3/2}}}\right)$

$\lim\limits_{x\to 0^+} \left(\frac{1}{x}\left(-\frac{2x^{3/2}}{1}\right)\right) = 0$

f) $\lim\limits_{x\to\infty} \left(\dfrac{x+1}{x+2}\right)^x =$ e^{-1}

$\ln y = \lim\limits_{x\to\infty} x \ln\left(\frac{x+1}{x+2}\right) =$

$= \lim\limits_{x\to\infty} \dfrac{\ln\left(\frac{x+1}{x+2}\right)}{\frac{1}{x}} = \frac{0}{0} \overset{L'H}{=}$

$\ln y = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x+1} - \frac{1}{x+2}}{-1/x^2} = \lim\limits_{x\to\infty} \dfrac{(x+2)-(x+1)}{(x+1)(x+2)}\left(\frac{-x^2}{1}\right)$

$\ln y = \lim\limits_{x\to\infty} \left(\frac{1}{x^2+3x+2} \cdot \frac{-x^2}{1}\right) = -1$

$y = e^{-1}$

$\int u\,dv = uv - \int v\,du$

2. Evaluate the integrals.

a) $\int x\cos 2x\,dx$

$u = x \qquad dv = \cos 2x\,dx$

$du = dx \qquad v = \frac{1}{2}\sin 2x$

$x\left(\frac{1}{2}\sin 2x\right) - \int \frac{1}{2}\sin 2x\,(dx)$

$\boxed{\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C}$

b) $\int \frac{\ln x}{x^2}dx$

$\ln x = u \qquad dv = x^{-2}\,dx$

$\frac{1}{x} = du \qquad v = \frac{1}{-1}x^{-1} = -\frac{1}{x}$

$= \ln x\left(-\frac{1}{x}\right) - \int -\frac{1}{x}\left(\frac{1}{x}\right)dx$

$-\frac{\ln x}{x} + \int x^{-2}\,dx =$

$-\frac{\ln x}{x} - \frac{1}{x} + C$

c) $\int \frac{x}{e^{3x}}dx$

$u = x \qquad dv = e^{-3x}$

$du = dx \qquad v = -\frac{1}{3}e^{-3x}$

$x\left(-\frac{1}{3}e^{-3x}\right) - \int -\frac{1}{3}e^{-3x}\,dx$

$-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}dx$

$\boxed{-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C}$

d) $\int \sin(\ln x)dx$ — let $m = \ln x$ so $e^m = x$

$e^m dm = dx$

$\int \underset{u}{\sin m}\,(\underset{dv}{e^m dm}) = \quad v = e^m$

$du = \cos m$

$e^m(\sin m) - \int e^m \cos m\,dm$

$\int \sin m(e^m dm) = e^m \sin m - \left[\int \underset{dv}{e^m}\,\underset{u}{\cos m}\,dm\right]$

$v = e^m \quad du = -\sin m$

$= e^m \sin m - \left[\cos m(e^m) - \int e^m(-\sin m)\right]$

$\int \sin m(e^m)dm = e^m \sin m - e^m\cos m + \int e^m \sin m\,dm$

$2\int \sin m\,e^m\,dm = e^m \sin m - e^m \cos m$

$= \frac{1}{2}\left[e^m \sin m - e^m \cos m\right]$

$= \frac{1}{2}\left[x\sin(\ln x) - x\cos(\ln x)\right] + C$

e) $\int \underset{u}{\cos 2x}\,\underset{dv}{(e^x)dx} = \cos 2x(e^x) + \int \underset{dv}{e^x}\,\underset{u}{(+2\sin 2x)}\,dx \qquad dv = e^x,\ v = e^x$

$du = -2\sin 2x \qquad v = e^x \qquad du = 4\cos 2x$

$\int \cos 2x(e^x)dx = e^x\cos 2x + \left[2(2\sin 2x)e^x - \int e^x(4\cos 2x)dx\right.$

$= e^x\cos 2x + 2e^x\sin 2x - 4\int e^x\cos 2x\,dx$

$5\int \cos 2x(e^x)\,dx = e^x\cos 2x + 2e^x\sin 2x$

$= \frac{1}{5}\left(e^x\cos 2x + 2e^x\sin 2x\right) + C$