MATH 152
Mrs. Bonny Tighe

QUIZ 3
7.7, 8.1
25 points

NAME Answers

Section ________ Wed. 2/22/06

1. Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

a) $\lim_{x\to 2} \frac{x^2-4}{x-2} =$ 4

$\lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = 4$

b) $\lim_{x\to\infty} \frac{x^2}{e^x} =$ 0

$\frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$

c) $\lim_{x\to 0} \frac{\sin^{-1} x}{x} =$ 1

$\frac{0}{0} \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1}$

$\lim_{x\to 0} \frac{1}{\sqrt{1-x^2}} = \frac{1}{1} = 1$

d) $\lim_{x\to\infty} \frac{(\ln x)^2}{x} =$ 0

$\frac{\infty}{\infty} \overset{L'H}{=} \left(\lim_{x\to\infty} \left(\frac{\ln x}{\sqrt{x}}\right)\right)^2$

$\left(\lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}\right)^2$

$\lim_{x\to\infty} \left(\frac{1}{x}\cdot\frac{2\sqrt{x}}{1}\right)^2 = \lim_{x\to\infty} \left(\frac{2}{\sqrt{x}}\right)^2 = 0$

e) $\lim_{x\to 0^+} \sin x \ln x$ = 0

$0\cdot\infty$

$\lim_{x\to 0^+} \frac{\ln x}{\csc x} = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc x \cot x} =$

$\lim_{x\to 0^+} \left(-\frac{1}{x(\csc x)\cot x}\right) = 0$

$\lim_{x\to 0^+} \left(-\frac{\sin x \tan x}{x}\right)$

f) $\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x =$ e^{-1}

$\ln y = \lim_{x\to\infty} x\left[\ln x - \ln(x+1)\right]$

$\ln y = \lim_{x\to\infty} \frac{\ln\left(\frac{x}{x+1}\right)}{\frac{1}{x}} = \frac{0}{0} \overset{L'H}{=}$

$\ln y = \lim_{x\to\infty} \left(\frac{\frac{1}{x} - \frac{1}{x+1}}{\frac{-1}{x^2}}\right) = \left(-\frac{x^2}{1}\right)$

$\lim_{x\to\infty} \left(\frac{\cancel{x}+1-\cancel{x}}{x(x+1)}\left(\frac{-x^2}{1}\right)\right) =$

$\lim_{x\to\infty} \left(\frac{-x^2}{x^2+x}\right) = e^{-1} = y$

$\int u\,dv = uv - \int v\,du$

$\int (\ln x)\left(\frac{1}{x^2}\right) dx$

$u = \ln x \quad dv = 1/x^2 = x^{-2}$

2. Evaluate the integrals.

a) $\int x \sin 3x dx$

$x = u \qquad \sin 3x = dv$

$dx = du \qquad -\frac{1}{3}\cos 3x = v$

$x\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x\,dx$

$-\frac{1}{3}x\cos 3x + \frac{1}{3}\cdot\frac{1}{3}\sin 3x + C$

$-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$

b) $\int \frac{\ln x}{x^2}dx$ $\quad du = \frac{1}{x} \quad v = \frac{1}{-1}x^{-1} = \frac{-1}{x}$

$\ln x\left(-\frac{1}{x}\right) - \int -\frac{1}{x}\left(\frac{1}{x}\right)dx$

$-\frac{1}{x}\ln x + \int x^{-2}dx =$

$-\frac{1}{x}\ln x - \frac{1}{x} + C$

c) $\int xe^{-2x}dx$

$u = x \qquad dv = e^{-2x}$

$du = dx \qquad v = -\frac{1}{2}e^{-2x}$

$-\frac{1}{2}xe^{-2x} - \int -\frac{1}{2}e^{-2x}dx$

$-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx$

$\boxed{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$

d) $\int \cos(\ln x)dx$ $\qquad w = \ln x$ so $x = e^w$

$dx = e^w dw$

$\int \underset{u}{\cos w}\,\underset{dv}{e^w dw} =$

$du = -\sin w \quad v = e^w$

$\int \cos w e^w dw = \cos w e^w - \int e^w(-\sin w)dw$

$= \cos w(e^w) + \int \underset{dv}{e^w}\,\underset{u}{\sin w}\,dw$

$v = e^w \quad du = \cos w$

$= e^w\cos w + \sin w(e^w) - \int e^w \cos w\,dw$

$2\int \cos w e^w dw = e^w(\cos w + \sin w)$

$= \frac{1}{2}e^w(\cos w + \sin w)$

$\int \cos w e^w dw = \boxed{\frac{1}{2}x\left[\cos(\ln x) + \sin(\ln x)\right] + C}$

e) $\int \underset{u}{e^{2x}}\,\underset{dv}{\sin x\,dx}$ $= 2e^{2x}(-\cos x) - \int -\cos x\,(2e^{2x})\,dx$

$du = 2e^{2x} \quad v = -\cos x$ $\qquad -2e^{2x}\cos x + 2\int \underset{dv}{\cos x}\,\underset{u}{e^{2x}}\,dx$

$v = \sin x \qquad du = 2e^{2x}$

$\int e^{2x}\sin x\,dx = -2e^{2x}\cos x + 2\left[e^{2x}\sin x - \int \sin x\,(2e^{2x})dx\right]$

$= -2e^{2x}\cos x + 2e^{2x}\sin x - 4\underline{\int \sin x\, e^{2x}dx}$

$5\int e^{2x}\sin x\,dx = -2e^{2x}\cos x + 2e^{2x}\sin x$

$\int e^{x}\sin x\,dx = \frac{1}{5}\left[-2e^{2x}\cos x + 2e^{2x}\sin x\right]$