MATH 152
Mrs. Bonny Tighe

QUIZ 2A

7.4-7.6
25 points

NAME Answers

Section__________ Wed. 2/15/06

1. Differentiate the following.

a) $y = \ln\sqrt{\dfrac{(x+1)(x^2+2)}{2x^2-3x+1}}$ $(2x-1)(x-1)$

$y = \frac{1}{2}[\ln(x+1) + \ln(x^2+2) - \ln(2x-1) - \ln(x-1)]$

$dy/dx = \frac{1}{2}\left[\frac{1}{x+1} + \frac{2x}{x^2+2} - \frac{2}{2x-1} - \frac{1}{x-1}\right]$

b) $y = 5^{\sqrt{x}}$

$dy/dx = \dfrac{5^{\sqrt{x}}\ln 5}{2\sqrt{x}}$

c) $f(x) = \cosh^3(\sinh^{-1}x)$

$f'(x) = 3\cosh^2(\sinh^{-1}x)(\sinh(\sinh^{-1}x))\left(\frac{1}{\sqrt{1+x^2}}\right)$

or $= \dfrac{3\cosh^2(\sinh^{-1}x)(x)}{\sqrt{1+x^2}}$

d) $f(x) = (\tanh^{-1}x^2)(\sqrt{x})$

$f'(x) = (\tanh^{-1}x^2)\left(\frac{1}{2\sqrt{x}}\right) + \sqrt{x}\left(\frac{1}{1-x^4}\right)(2x)$

$= \dfrac{\tanh^{-1}x^2}{2\sqrt{x}} + \dfrac{2x\sqrt{x}}{1-x^4}$

2. Find the numerical value of each expression:

a) $\sin(\underbrace{\cos^{-1}\sqrt{3}/2}_{\pi/6}) =$ $1/2$

b) $\sinh(0) =$ 0

$\dfrac{e^x - e^{-x}}{2}$

c) $\cos(\tan^{-1}x) =$ $\dfrac{1}{\sqrt{1-x^2}}$

3. Find the equation of the tangent line to the curve y=ln(lnx) at the point (e,0).

$m = dy/dx$

$dy/dx = \frac{1}{\ln x}\cdot\frac{1}{x}$ at $x = e$

$\frac{1}{\ln e}\cdot\frac{1}{e} = \frac{1}{e}$

$y - y_1 = m(x - x_1)$

$y - 0 = (\quad)(x - e)$

$y = 1/e\,(x - e)$

$y = 1/e\, x - 1$

$$-\int \cos^{-1}x \cdot \frac{-1}{\sqrt{1-x^2}}\,dx$$

4. Evaluate:

a) $\int_1^2 \frac{2+t^2}{t^3}\,dt =$ $3/4 + \ln 2$

$$\int_1^2 2t^{-3} + \frac{1}{t}\,dt$$

$$\frac{2}{-2}t^{-2} + \ln t \Big|_1^2 =$$

$$\frac{-1}{t^2} + \ln t \Big|_1^2 = \left(\frac{-1}{4} + \ln 2\right) - (-1 + \ln 1)$$

$$-\frac{1}{4} + \ln 2 + 1 - 0$$

$$3/4 + \ln 2$$

b) $\int \frac{\cos^{-1}x}{\sqrt{1-x^2}}\,dx =$ ________

$$-\int u'\,du = -\frac{1}{2}u^2 + C$$

$$-1/2(\cos^{-1}x)^2 + C$$

c) $\int \frac{\sec^2 x}{1+\tan^2 x}\,dx =$ ________

$$\int \frac{\sec^2 x}{\sec^2 x}\,dx = \int 1\,dx$$

$$x + C$$

or $u = \tan x$, $du = \sec^2 x\,dx$

$$\int \frac{du}{1+u^2} = \tan^{-1}u + C =$$

$$\tan^{-1}(\tan x) + C = x + C$$

d) $\int \frac{e^x}{1-e^x}\,dx =$ ________

$u = 1 - e^x$

$du = -e^x\,dx$

$$-\int \frac{1}{u}\,du$$

$$-\ln u + C = \cancel{\ln t}$$

$$-\ln|1-e^x| + C$$

e) $\int x\, 3^{x^2}\,dx =$ $\frac{3^{x^2}}{2\ln 3} + C$

$$\frac{1}{2}\int 3^u\,du = \frac{1}{2}\frac{3^u}{\ln 3} + C$$

f) $\int \frac{1}{3x\sqrt{9x^2-1}}\,3\,dx =$ ________

$u = 3x$

$du = 3\,dx$

$$\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}u + C = \sec^{-1}(3x) + C$$

5) Use logarithmic differentiation to find the derivative of

$$f(x) = (1+\sin x)^{x^3}$$

$$\ln y = x^3 \ln(1+\sin x)$$

$$\frac{1}{y}\,dy/dx = x^3 \cdot \frac{1}{1+\sin x}(\cos x) + \ln(1+\sin x)\,3x^2$$

$$dy/dx = \left(\frac{x^3\cos x}{1+\sin x} + 3x^2\ln(1+\sin x)\right)(1+\sin x)^{x^3}$$