MATH 152
Mrs. Bonny Tighe

QUIZ 2

7.4-7.6
25 points

NAME Answers

Section__________ Wed. 2/15/06

1. Differentiate the following.

a) $y = \ln\sqrt{\dfrac{(x+1)(x^2+2)}{2x^2-3x+1}}$ $(2x-1)(x-1)$

$y = \frac{1}{2}\left[\ln(x+1) + \ln(x^2+2) - \ln(2x-1) - \ln(x-1)\right]$

$dy/dx = \frac{1}{2}\left[\frac{1}{x+1} + \frac{1}{x^2+2}(2x) - \frac{1}{2x-1}(2) - \frac{1}{x-1}\right] =$

$= \frac{1}{2}\left[\frac{1}{x+1} + \frac{2x}{x^2+2} - \frac{2}{2x-1} - \frac{1}{x-1}\right]$

b) $y = 3^{\sqrt{x}}$

$dy/dx = 3^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \ln 3 =$

$\frac{(\ln 3)\,3^{\sqrt{x}}}{2\sqrt{x}}$

c) $f(x) = \sinh^3(\tanh^{-1} x)$

$f'(x) = 3\sinh^2(\tanh^{-1} x)(\cosh(\tanh^{-1})(x))\left(\frac{1}{1-x^2}\right)$

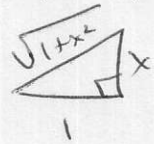
d) $f(x) = (\cosh^{-1} x^2)(\sqrt{x})$

$f'(x) = \cosh^{-1} x^2\left(\frac{1}{2\sqrt{x}}\right) + \sqrt{x}\left(\frac{1}{\sqrt{x^4-1}}(2x)\right)$

$f'(x) = \frac{\cosh^{-1} x^2}{2\sqrt{x}} + \frac{2x\sqrt{x}}{\sqrt{x^4-1}}$

2. Find the numerical value of each expression:

a) $\tan(\cos^{-1}\sqrt{3}/2) = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ b) $\cosh(0) = 1$ $\frac{e^x+e^{-x}}{2} =$ c) $\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$



3. Find the equation of the tangent line to the curve y=ln(lnx) at the point (e,0).

$y - y_1 = m(x - x_1)$

$y - 0 = 1/e\,(x - e)$

$y = \frac{1}{e}x - 1$

$m = dy/dx = \frac{1}{\ln x} \cdot \frac{1}{x}$ at $x = e$

$\frac{1}{\ln e} \cdot \frac{1}{e} = \frac{1}{1} \cdot \frac{1}{e} = \frac{1}{e}$

4. Evaluate:

a) $\int_1^2 \frac{4+u^2}{u^3}\,du = \frac{3}{2} + \ln 4$

$\int_1^2 \left(4u^{-3} + \frac{1}{u}\right) du =$

$\frac{4}{-2}u^{-2} + \ln|u| \Big|_1^2$

$-\frac{2}{u^2} + \ln u \Big|_1^2 =$

$\left(-\frac{2}{4} + \ln 4\right) - \left(-\frac{2}{1} + \ln 1\right)$

$-\frac{1}{2} + \ln 4 + 2 + 0 =$

b) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx =$ ________

$(\sin^{-1}x)' = \frac{1}{\sqrt{1-x^2}}\,dx$

$\int u'\,du = \frac{1}{2}u^2 + C$

$\frac{1}{2}(\sin^{-1}x)^2 + C$

c) $\int \frac{\cos x}{1+\sin^2 x}\,dx =$ ________

$u = \sin x$
$du = \cos x\,dx$

$\int \frac{du}{1+u^2}$

$\tan^{-1} u + C$

$\tan^{-1}(\sin x) + C$

d) $\int \frac{e^x}{1-e^{2x}}\,dx =$ ________

$u = e^x$
$du = e^x dx$

$\int \frac{1}{1-u^2}\,du = \tanh^{-1}(u) + C$

$\tanh^{-1}(e^x) + C$

e) $\int x\, 2^{x^2}\,dx =$ ________

$u = x^2$
$du = 2x\,dx$

$\frac{1}{2}\int 2^u\,du$

$\frac{1}{2} \cdot \frac{2^u}{\ln 2} + C$

$\frac{2^{x^2}}{2\ln 2} + C$ or ~~$\frac{2^{x^2-1}}{\ln 2} + C$~~

f) $\int \frac{2}{2x\sqrt{4x^2-1}}\,dx =$ ________

$u = 2x$
$du = 2dx$

$\int \frac{du}{u\sqrt{u^2-1}} =$

$\sec^{-1} u + C$

$\sec^{-1}(2x) + C$

5) Use logarithmic differentiation:

$f(x) = (1+x^2)^{\sin x}$

$\ln y = \sin x \ln(1+x^2)$

$\frac{1}{y}\,dy/dx = \sin x\left(\frac{2x}{1+x^2}\right) + \ln(1+x^2)(\cos x)$

$dy/dx = \left[\frac{2x\sin x}{1+x^2} + \cos x \ln(1+x^2)\right](1+x^2)^{\sin x}$