$1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \cdots$

MATH 152 **QUIZ 10A** NAME Answers

Mrs. Bonny Tighe 25 points

12.11 Section ______ Wed. 5/3/06

1. Expand each of the following as a power series using the binomial series. State the radius and interval of convergence. a) $\frac{x}{\sqrt{1+x}}$ $= x(1+x)^{-1/2}$ $k = -1/2$

$x\left(1 + \sum_{n=1}^{\infty} \frac{x^n}{n!}\right)\frac{(-1)^n}{2^n}$

$1 + -\frac{1}{2} + -\frac{1}{2}(-\frac{3}{2}) + (-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2}) +$

$1 \quad +1 \quad +1.3 \quad +1.3.5 \quad +1.3.5.7$

$n=4 \quad (2n-1)$

$$\boxed{x + \sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}(1.3.5\ldots(2n-1))}{n!\,2^n}}$$

$$\lim_{n\to\infty}\left|\frac{x^{n+2}(1.3.5\ldots(2n-1)(2n+1))}{(n+1)!\,2^{n+1}} \cdot \frac{n!\,2^n}{x^{n+1}(1.3.5\ldots(2n-1))}\right| = \lim_{n\to\infty}\left|\frac{x(2n+1)}{(n+1)2}\right| = |x|$$

$$\boxed{R = 1 \quad I = [-1, 1]}$$

$x = -1 \quad \sum_{n=1}^{\infty} \frac{(-1)^n(-1)^{n+1}(1.3.5\ldots(2n-1))}{n!\,2^n}$ AST, convergent

$x = 1 \quad \sum_{n=1}^{\infty} \frac{(-1)^n(1.3.5\ldots(2n-1))}{n!\,2^n}$ AST, convergent

b) $\frac{1}{(4-x)}$ $= \frac{1/4}{(1+(-\frac{x}{4}))}$ $= \frac{1}{4}\left(1+(-\frac{x}{4})\right)^{-1}$ $k = -1$

$1 + -1 + -1(-2) + -1(-2)(-3) + (-1)(-2)(-3)(-4)$

$\frac{1}{4}\left[1 + \sum_{n=1}^{\infty} \frac{(-\frac{x}{4})^n}{n!}\right.$

$\frac{1}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n x^n (-1)^n \cancel{n!}}{4^{n+1}\,n!} =$ $\boxed{\frac{1}{4} + \sum_{n=1}^{\infty} \frac{x^n}{4^{n+1}}}$

$\lim_{n\to\infty}\left|\frac{x^{n+1}}{4^{n+2}} \cdot \frac{4^{n+1}}{x^n}\right| = \left|\frac{x}{4}\right| < 1$ $\boxed{R = 4 \quad I = (-4, 4)}$

$x = -4 \quad \sum_{n=1}^{\infty} \frac{(-4)^n}{4^{n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{4^{n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{4}$ divergent by test for divergence

$x = 4 \quad \sum_{n=1}^{\infty} \frac{4^n}{4^{n+1}} = \sum_{n=1}^{\infty} \frac{1}{4}$ divergent by test for divergence

$$\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}} = (1+(-x^2))^{-1/2}$$

$k = -1/2$

2. Use the binomial series to expand a Maclaurin series for $\sin^{-1} x$

$$1 + \sum_{n=1}^{\infty} \frac{(-x^2)^n}{n!} \frac{(-1)^n}{2^n}$$

$$1 + -\tfrac{1}{2} + -\tfrac{1}{2}(-\tfrac{3}{2}) + (-\tfrac{1}{2})(-\tfrac{3}{2})(-\tfrac{5}{2}) +$$

$$1 + \frac{1}{2} + \frac{1\cdot3}{2^2} + \frac{1\cdot3\cdot5}{2^3} \cdots (2n-3)$$

$n=4$

$$\int \left(1 + \sum_{n=1}^{\infty} \frac{x^{2n}(1\cdot3\cdot5\cdots(2n-3))}{n!\,2^n}\right) dx =$$

$$\boxed{C + x + \sum_{n=1} \frac{(1\cdot3\cdot5\cdots(2n-3))\,x^{2n+1}}{(2n+1)\,n!\,2^n}}$$

3. Use the binomial series to expand $\sqrt[4]{1-x}$ as a power series and use this to estimate $\sqrt[4]{\cancel{2.98}}$. $\sqrt[4]{.98}$

$(1+(-x))^{1/4}$ $k = 1/4$

$$1 + \frac{1}{4}(-x) + \frac{\frac{1}{4}\left(-\frac{3}{4}\right)(-x)^2}{2!} + \frac{\frac{1}{4}\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)(-x)^3}{3!} + \cdots$$

$$1 - \frac{1}{4}x - \frac{3}{32}x^2 - \frac{21}{64(\cancel{6})_2}x^3 -$$

$$1 - \frac{1}{4}\left(\frac{2}{100}\right) - \frac{3}{32}\left(\frac{2}{100}\right)^2 - \frac{7}{128}\left(\frac{2}{100}\right)^3 -$$

$$1 - \frac{1}{200} - \frac{3}{80000} -$$