

1. Find $(f^{-1})'(a)$ for $f(x) = \frac{x+1}{5-x}$ at $a = 1$.

$$\frac{x+1}{5-x} = 1$$

$$x+1 = 5-x$$

$$2x = 4$$

$$x = 2$$

$$f(2) = 1 \text{ so } g(1) = 2$$

$$f'(x) = \frac{(5-x) - (x+1)(-1)}{(5-x)^2}$$

$$f'(x) = \frac{5-x+x+1}{(5-x)^2} = \frac{6}{(5-x)^2}$$

$$g'(a) = \frac{1}{f'(g(a))}$$

$$g'(1) = \frac{1}{f'(2)} = \frac{1}{\frac{6}{(5-2)^2}} = \frac{1}{\frac{6}{9}} = \frac{9}{6} = \frac{3}{2}$$

2. Find the limit.

a) $\lim_{x \rightarrow \infty} (-e^{-x} + 1) = 1$

b) $\lim_{x \rightarrow \infty} \frac{2e^x - e^{-x}}{e^x + e^{-x}} = 2$

$\lim_{x \rightarrow \infty} (1.000001)^x = \infty$

3. Find dy/dx if $e^{xy} = x + \cos y$

$$e^{xy} (x \frac{dy}{dx} + y(1)) = 1 + -\sin y \frac{dy}{dx}$$

$$x e^{xy} \frac{dy}{dx} + \sin y \frac{dy}{dx} = 1 - e^{xy} (y)$$

$$\frac{dy}{dx} = \frac{1 - y e^{xy}}{x e^{xy} + \sin y}$$

4. Find a formula for the inverse of the function $f(x) = \frac{2-3^x}{1+3^x}$

$$x = \frac{2-3^y}{1+3^y}$$

$$x + x 3^y = 2 - 3^y$$

$$x 3^y + 3^y = 2 - x$$

$$3^y (x+1) = 2-x$$

$$3^y = \frac{2-x}{x+1}$$

$$\log_3 \left(\frac{2-x}{x+1} \right) = y$$

$$\text{or } \log_3(2-x) - \log_3(x+1) = y$$

$$y \ln 3 = \ln \left(\frac{2-x}{x+1} \right)$$

$$y = \ln \left(\frac{2-x}{x+1} \right) \cdot \frac{1}{\ln 3} \text{ or } y = \log_3 \left(\frac{2-x}{x+1} \right)$$

5. Find the derivative: a) $y = 2e^{x^3}(x\sqrt{x}) = 2e^{x^3} \cdot x^{3/2}$

$$\frac{dy}{dx} = 2e^{x^3} \cdot \frac{3}{2}x^{1/2} + x^{3/2} \cdot 2e^{x^3}(3x^2) =$$

$$3\sqrt{x} e^{x^3} + 6x^3\sqrt{x} e^{x^3}$$

b) $g(x) = \frac{e^{\cos x}}{1-2e^{2x}}$

$$g'(x) = \frac{(1-2e^{2x})(e^{\cos x}(-\sin x)) - e^{\cos x}(-4e^{2x})}{(1-2e^{2x})^2}$$

$$g'(x) = \frac{-\sin x e^{\cos x}(1-2e^{2x}) + 4e^{2x} e^{\cos x}}{(1-2e^{2x})^2}$$

6. Evaluate the integral:

a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

$$2 \int e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx$$

$$2e^{\sqrt{x}} \Big|_1^4$$

$$2e^2 - 2e^1$$

$$\int \sin u du$$

b) $\int e^{3x} \sin(e^{3x}) dx$

$$-\frac{1}{3} \cos e^{3x} + C$$

c) $\int \frac{e^{\sin x}}{\sec x} dx$

$$\int e^u du$$

$$\int e^{\sin x} \cos x dx$$

$$e^{\sin x} + C$$

7. Solve for x:

a) $2e^{x+1} = 6$

$$e^{x+1} = 3$$

$$\ln 3 = x+1$$

$$\ln 3 - 1 = x$$

b) $e^{2x} - e^x = 2$

$$e^{2x} - e^x - 2 = 0$$

$$(e^x - 2)(e^x + 1) = 0$$

$$e^x = 2, \neq$$

$$\ln 2 = x$$

c) $\ln x - \ln(x+2) = 3$

$$\ln \frac{x}{x+2} = 3$$

$$\frac{x}{x+2} = e^3$$

$$x = e^3 x + 2e^3$$

$$x - xe^3 = 2e^3$$

$$x(1-e^3) = 2e^3$$

$$x = \frac{2e^3}{1-e^3}$$