MATH 152
Mrs. Bonny Tighe

EXAM III
100 points
12.5-11.4

NAME Answers

Section ________ Wed. 5/10/06

There are 11 problems worth 10 points each.

1. Test the series for convergence or divergence. State and show the test.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$$

Comparison Test

$\frac{1}{\sqrt{n}}$ is divergent by p-series $n<1$

$\frac{\ln n}{\sqrt{n}} \geq \frac{1}{\sqrt{n}}$ so both are divergent

Alternating Series Test: $\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \frac{\infty}{\infty} \overset{L'H}{=} \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} =$

$\lim_{n\to\infty} \frac{2}{\sqrt{n}} = 0$ so $\boxed{\text{conditionally convergent}}$

2, Test each series for divergence or convergence. State and show the test.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cos n}{(2n)!}$$

Ratio Test

$\lim_{n\to\infty} \left| \frac{1}{(2n)!(2n+2)} \cdot \frac{(2n)!}{1} \right| =$

$\lim_{n\to\infty} \left| \frac{1}{2n+2} \right| = 0 < 1$

So convergent

Comparison Test

$\cos n \leq 1$ so

$\frac{\cos n}{(2n)!} \leq \frac{1}{(2n)!}$ $\quad \frac{1}{(2n)!}$ is convergent

so $\frac{\cos n}{(2n)!}$ is also convergent

$\boxed{\text{absolutely convergent}}$

3. Find the radius of convergence and the interval of convergence of the series.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n (3x-2)^n}{2(n+1)!}$

$$\lim_{n\to\infty} \left| \frac{(3x-2)^{n+1}}{2(n+1)!\,(n+2)} \cdot \frac{2(n+1)!}{(3x-2)^n} \right| = \left| \frac{3x-2}{n+2} \right| = 0 < 1 \text{ always}$$

$$\boxed{\begin{array}{l} \text{so } R = \infty \\ I = (-\infty, \infty) \end{array}}$$

b) $\sum_{n=0}^{\infty} \sqrt{n}(x-1)^n$

$$\lim_{n\to\infty} \left| \frac{\sqrt{n+1}\,(x-1)^{n+1}}{\sqrt{n}\,(x-1)^n} \right| = \lim_{n\to\infty} \left| \sqrt{\frac{n+1}{n}}\,(x-1) \right| = |x-1| < 1$$

$$\boxed{\begin{array}{l} R = 1 \\ I = (0, 2) \end{array}}$$

$\underline{x=0}$ $\sum_{n=0}^{\infty} \sqrt{n}(-1)^n$ divergent, does not head to 0

$\underline{x=2}$ $\sum_{n=0}^{\infty} \sqrt{n}(1)^n$ – divergent, does not head towards 0

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

4. Find a power series representation for the function and determine the radius ~~and~~ ~~interval~~ of convergence. $f(x) = \dfrac{1}{4-x^3}$ $= \dfrac{1/4}{1-\frac{x^3}{4}}$

$$\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{x^3}{4}\right)^n = \sum_{n=0}^{\infty} \frac{x^{3n}}{4^{n+1}}$$

$$\lim_{n\to\infty}\left|\frac{x^{3n+3}}{4^{n+2}}\cdot\frac{4^{n+1}}{x^{3n}}\right| = \lim_{n\to\infty}\left|\frac{x^3}{4}\right| = \left|\frac{x^3}{4}\right| < 1$$

$$R = \sqrt[3]{4}$$

5. Evaluate the indefinite integral as an infinite series. $\int \dfrac{\cos x - e^{\sqrt{x}}}{x^2}\,dx$

From the text: $\cos x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$

$$\int \frac{1}{x^2}\sum_{n=0}^{\infty}\left(\frac{(-1)x^{2n}}{(2n)!} + \frac{(\sqrt{x})^n}{n!}\right) = \int\sum_{n=0}^{\infty}\left(\frac{(-1)^n x^{2n-2}}{(2n)!} + \frac{x^{n/2-2}}{n!}\right)$$

$$C + \sum_{n=0}^{\infty}\left(\frac{(-1)^n x^{2n-1}}{(2n-1)(2n)!} + \frac{x^{n/2-1}}{\left(\frac{n}{2}-1\right)n!}\right)$$

6. Find the Taylor series for f(x) centered at the given value of $f(x) = \sqrt{x}$ at a = 4 and find the radius of convergence.

	at $a=4$
$f(x) = x^{1/2}$	2
$f'(x) = \frac{1}{2}x^{-1/2}$	$\frac{1}{2} \cdot \frac{1}{2}$
$f''(x) = -\frac{1}{4}x^{-3/2}$	$-\frac{1}{4} \cdot \frac{1}{2^3}$
$f'''(x) = \frac{1\cdot 3}{2^3}x^{-5/2}$	$\frac{1\cdot 3}{2^3} \cdot \frac{1}{2^5}$
$f''''(x) = \frac{-1\cdot 3\cdot 5}{2^4}x^{-7/2}$	$\frac{-1\cdot 3\cdot 5}{2^4} \cdot \frac{1}{2^7}$

$$\underset{n=0}{2} + \underset{n=1}{\frac{1}{\cancel{x}^2}} + \underset{n=2}{\frac{1}{2^5 \cdot 2}} + \underset{n=3}{\frac{1\cdot 3}{2^8}} + \underset{n=4}{\frac{1\cdot 3\cdot 5}{2^{11}}}$$

$$f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots$$

$$2 + \frac{1}{4}(x-4) + \sum_{n=2}^{\infty} \frac{(x-4)^n}{n!} \frac{(-1)^{n+1}(1\cdot 3\cdot 5 \cdots (2n-3))}{2^{3n-1}}$$

$$\boxed{2 + \frac{x-4}{4} + \sum_{n=2}^{\infty} \frac{(-1)^{n+1}(x-4)^n(1\cdot 3\cdot 5\cdots(2n-3))}{n!\; 2^{3n-1}}}$$

$$\lim_{n\to\infty} \left| \frac{(x-4)^{n+1}(1\cdot 3\cdot 5\cdots(2n-3)(2n-1))}{(n+1)!\; 2^{3n}} \cdot \frac{n!\; 2^{3n-1}}{(x-4)^n(1\cdot 3\cdot 5\cdots(2n-3))} \right| =$$

$$\lim_{n\to\infty} \left| \frac{(x-4)(2n-1)}{(n+1)(2)} \right| = |x-4| < 1 \qquad R = 1$$

7. Find the Maclaurin series of f(x) and its radius of convergence. $f(x) = \frac{1}{(x+2)^2}$

$f(x) = (x+2)^{-2}$ @ $x=0$ $\frac{1}{4}$

$f'(x) = -2(x+2)^{-3}$ $-2\left(\frac{1}{8}\right)$

$f''(x) = 6(x+2)^{-4}$ 6 $\frac{1}{16}$

$f'''(x) = -4!\,(x+2)^{-5}$ $\frac{-4!}{2^5}$

$\underset{n=0}{\frac{1}{4}} + \underset{n=1}{\frac{2}{2^3}} + \underset{n=2}{\frac{3!}{2^4}} + \underset{n=3}{\frac{4!}{2^5}}$

$f(0) + \frac{f'(0)\,x}{1!} + \frac{f''(0)\,x^2}{2!} + \frac{f'''(0)\,x^3}{3!} + \frac{f^{(4)}(0)\,x^4}{4!} + \cdots$

$\frac{1}{4} + \sum_{n=1}^{\infty} \frac{x^n(-1)^n}{n!} \frac{(n+1)!}{2^{n+2}} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^n (n+1)}{2^{n+2}}}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}(n+2)}{2^{n+2}} \cdot \frac{2^{n+1}}{x^n(n+1)} \right| = \lim_{n\to\infty} \left| \frac{x}{2}\left(\frac{n+2}{n+1}\right) \right| = \left|\frac{x}{2}\right| < 1$

$R = 2$

$(1+x)^k$

8. Expand $\dfrac{1}{(2-x)^3}$ as a power series using the binomial series. State the radius ~~and interval~~ of convergence.

$$(1+x)^k = 1 + \frac{kx}{1!} + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \cdots$$

$$f(x) = (2-x)^{-3} = \frac{1}{8}\left(1+\left(-\frac{x}{2}\right)\right)^{-3} \qquad k=-3$$

$$\frac{1}{8}\left(1 + \sum_{n=1}^{\infty} \frac{\left(\frac{-x}{2}\right)^n}{n!} \frac{(-1)^n(n+2)!}{2}\right) = \boxed{\frac{1}{8} + \sum_{n=1}^{\infty} \frac{x^n(n+1)(n+2)}{2^{n+4}}}$$

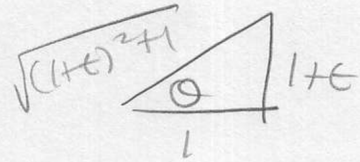
$$1 + \frac{2(-3)}{2} + \frac{2(-3)(-4)}{2} + \frac{2(-3)(-4)(-5)}{2} + \cdots$$

$$\underset{n=0}{1} + \underset{n=1}{\frac{3!}{2}} + \underset{n=2}{\frac{4!}{2}} + \frac{5!}{2} + \cdots$$

$$\lim_{n\to\infty}\left|\frac{x^{n+1}\cancel{(n+2)}(n+3)}{2^{n+5}} \cdot \frac{2^{n+4}}{x^n(n+1)\cancel{(n+2)}}\right| =$$

$$\lim_{n\to\infty}\left|\frac{x}{2}\left(\frac{n+3}{n+1}\right)\right| = \left|\frac{x}{2}\right| < 1 \qquad R=2$$

$dy/dt = \frac{1}{1+t}$ $\qquad \frac{dx}{dt} = -\frac{1}{(1+t)^2}$



9. Find the length of the curve $x = \frac{1}{1+t}$ *and* $y = \ln(1+t)$ $\quad 0 \le t \le 2$

$\int_0^2 \sqrt{\left(\frac{1}{1+t}\right)^2 + \left(\frac{-1}{(1+t)^2}\right)^2}\, dt = \int_0^2 \frac{\sqrt{(1+t)^2+1}}{(1+t)^2}\, dt$ $\qquad \tan\theta = 1+t$

$\sec^2\theta\, d\theta = dt$

$\int_0^2 \frac{\sqrt{\tan^2\theta+1}\ \sec^2\theta\, d\theta}{\tan^2\theta} = \int_0^2 \frac{\sec^3\theta}{\tan^2\theta}\, d\theta = \int_0^2 \csc^2\theta \sec\theta\, d\theta$

$dv \qquad u$

$du = \sec\theta\tan\theta\, d\theta$

$v = -\cot\theta$

$-\sec\theta\cot\theta - \int -\cot\theta \sec\theta \cancel{\tan\theta}\, d\theta =$

$-\sec\theta\cot\theta + \int \sec\theta\, d\theta = -\sec\theta\cot\theta + \ln|\sec\theta + \tan\theta|\,\Big|_0^2 =$

$-\sqrt{(1+t)^2+1}\left(\frac{1}{1+t}\right) + \ln\left|\sqrt{(1+t)^2+1} + (1+t)\right|\,\Big|_0^2 =$

$\left(-\frac{\sqrt{10}}{3} + \ln|\sqrt{10}+3|\right) - \left(-\sqrt{2}\left(\frac{1}{1}\right) + \ln|\sqrt{2}+1|\right) =$

$-\frac{\sqrt{10}}{3} + \ln|\sqrt{10}+3| + \sqrt{2} - \ln|\sqrt{2}+1|$

10. a) Find a Cartesian equation for the curve described by the polar equation
$r = \tan\theta \sec\theta$

$r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$ $\qquad r\cos^2\theta = \sin\theta$ $\qquad x^2 = y$

$r^2\cos^2\theta = r\sin\theta$

b) Convert the Cartesian coordinates (-2, 3) to polar coordinates.

$r^2 = x^2 + y^2 = 4+9$ $\qquad r = \sqrt{13}$ $\qquad \left(-\sqrt{13},\ \tan^{-1}(-3/2)\right)$

$\tan\theta = 3/-2$

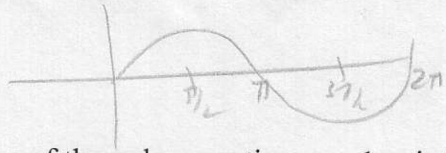
c) Convert the polar coordinates ($(3, \frac{5\pi}{6})$ to Cartesian coordinates.

$x = r\cos\theta$ $\qquad y = r\sin\theta$

$x = 3\cos 5\pi/6$ $\qquad = 3\sin 5\pi/6$

$3(-\sqrt{3}/2)$ $\qquad 3(1/2)$

$\left(-3\sqrt{3}/2,\ 3/2\right)$



11. Sketch the curve of the polar equation $r = 1 + \sin 2\theta$

θ	$1+\sin 2\theta$
0	1
$\pi/6$	$\sqrt{3}/2 + 1$
$\pi/4$	2
$\pi/3$	$\sqrt{3}/2 + 1 < 2$
$\pi/2$	1
$2\pi/3$	$-\sqrt{3}/2 + 1$
$3\pi/4$	0
$5\pi/6$	$-\sqrt{3}/2 + 1$
π	1
$7\pi/6$	$\sqrt{3}/2 + 1$
$5\pi/4$	2
$4\pi/3$	$\sqrt{3}/2 + 1$
$3\pi/2$	1
$5\pi/3$	$-\sqrt{3}/2 + 1$
$7\pi/4$	0
$11\pi/6$	$-\sqrt{3}/2 + 1$
2π	1

$7\pi/2$

$11\pi/3$