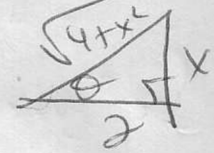
MATH 152 **EXAM IIA** NAME Answers

Mrs. Bonny Tighe 100 points

8.4-12.4 Section ________ Wed. 4/5/2006

There are 11 problems worth 10 points each



1. a) Evaluate the integrals : a) $\int \sqrt{4+x^2}\ dx$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$\int \sqrt{4+4\tan^2\theta}\, 2\sec^2\theta\, d\theta$

$4\int \sec^3\theta\, d\theta = 4\int \underset{u}{\sec\theta}\ \underset{dv}{\sec^2\theta\, d\theta} = \sec\theta\tan\theta - \int \tan\theta\sec\theta\tan\theta\, d\theta$

$v = \tan\theta$

$du = \sec\theta\tan\theta$

$4\int \sec^3\theta\, d\theta = 4\left[\sec\theta\tan\theta - \int \sec\theta \underset{(\sec^2\theta - 1)}{\tan^2\theta}\, d\theta\right]$

$= 4\sec\theta\tan\theta - 4\int \sec^3\theta\, d\theta + 4\int \sec\theta\, d\theta$

$8\int \sec^3\theta\, d\theta = 4\sec\theta\tan\theta + 4\ln|\sec\theta + \tan\theta| + C$

$= \frac{1}{2}\left(\frac{\sqrt{4+x^2}}{2}\right)\left(\frac{x}{2}\right) + \frac{1}{2}\ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$

b) $\int \frac{e^x}{e^{2x}+3e^x+2}\, dx = \int \frac{du}{(u+2)(u+1)} = \int \frac{-1}{u+2}\, du + \int \frac{1}{u+1}\, du$

$u = e^x$

$du = e^x dx$

$\frac{A}{u+2} + \frac{B}{u+1} = \frac{1}{(u+2)(u+1)}$

$A(u+1) + B(u+2) = 1$

$u = -1 \qquad B = 1$

$u = -2 \qquad A = -1$

$-\ln|u+2| + \ln|u+1| + C \quad = \ln|e^x+1| - \ln|e^x+2| + C$

2. Evaluate the integral. $\int e^{-x} \cos x \, dx$

$u \quad dv \qquad du = -e^{-x} \quad v = \sin x$

$\int e^{-x} \cos x dx = e^{-x}(\sin x) - \int \sin x(-e^{-x})dx$

$= \sin x e^{-x} + \int e^{-x} \sin x dx$

$u \quad dv \qquad du = -e^{-x} + \quad = -\cos x$

$= \sin x e^{-x} + e^{-x}(-\cos x) - \int -\cos x(-e^{-x})dx$

$2\int e^{-x} \cos x dx = \sin x e^{-x} - \cos x e^{-x}$

$= \frac{1}{2}\left[\sin x e^{-x} - \cos x e^{-x}\right]$

3. Determine whether each improper integral is divergent or convergent and evaluate those that are convergent using improper integrals.

a) $\int_0^{\infty} \frac{1}{3}e^{-3x} \, dx$

b) $\int_{-1}^{1} \frac{1}{(x+1)^3} dx \qquad (x+1)^{-3}$

$\lim_{t \to \infty} \int_0^t \frac{1}{3} e^{-3x} dx$

$\lim_{t \to \infty} -\frac{1}{9} e^{-3x} \Big|_0^t =$

$\lim_{t \to \infty} \left(-\frac{1}{9} e^{-3t} + \frac{1}{9} e^{-0}\right)$

$\lim_{t \to \infty} \left(-\frac{1}{9e^{3t}} + \frac{1}{9}\right) = \frac{1}{9}$

So Convergent

$\lim_{t \to -1} \int_t^1 (x+1)^{-3} dx$

$\lim_{t \to -1} \frac{1}{-2}(x+1)^{-2} \Big|_t^1 =$

$\lim_{t \to -1} \left(-\frac{1}{2(2)^2} + \frac{1}{2}\left(\frac{1}{(t+1)^2}\right)\right)$

DNE

So divergent

$dy/dx = \frac{1}{\sec x} \cdot \sec x \tan x$ d

4. Find the arc length of the curve $y = \ln(\sec x)$ *on the* int *erval* $0 \le x \le \pi/4$.

$$\int_0^{\pi/4} \sqrt{1 + (dy/dx)^2}\, dx = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx =$$

$$\int_0^{\pi/4} \sec x \, dx = \ln|\sec x + \tan x| \Big|_0^{\pi/4} =$$

$$\ln|\sec \pi/4 + \tan \pi/4| - \ln|\sec 0 + \tan 0| =$$

$$\ln|\sqrt{2} + 1| - \ln|1 + 0| = \ln|\sqrt{2}+1|$$

5. Find the sum of the following convergent series.

a) $\sum_{n=1}^{\infty} e^{-n} 2^{n+1}$ $\quad \frac{2^{n+1}}{e^n}$

$$\sum_{n=1}^{\infty} 2\left(\frac{2}{e}\right)^n =$$

$$\sum_{n=1}^{\infty} \frac{4}{e}\left(\frac{2}{e}\right)^{n-1}$$

$$\text{Sum} = \frac{a}{1-r}$$

$$\frac{4/e}{1 - 2/e} = \frac{4}{e-2}$$

b) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n + 2}$

$(n+2)(n+1)$

$A(n+1) + B(n+2) = 1$

$n = -1 \quad B = 1$

$n = -2 \quad A = -1$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$\frac{1}{2} + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \frac{1}{5} + \cdots$$

$$-\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} - \frac{1}{5} + \cdots$$

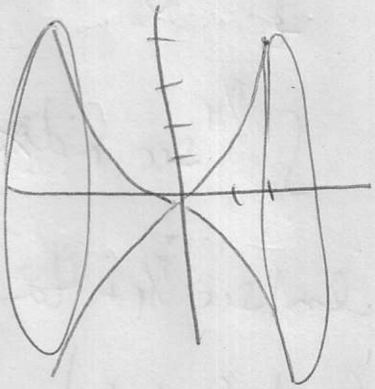
$\text{Sum} = 1/2$

$\sqrt{y} = y$

6. SET UP BUT DO NOT EVALUATE the area of the surface generated by revolving the given curve about the x-axis and sketch the figure. $y = x^2$, $0 \le y \le 4$

$$2\pi \int_c^d y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$2\pi \int_0^4 y \sqrt{1 + \left(\frac{1}{2\sqrt{y}}\right)} dy$$



$$\text{or } \cancel{4}\pi \int_0^2 x^2 \sqrt{1 + (2x)^2}\, dx$$

7. State and use the Integral Test to determine whether the series is convergent or divergent.

a) $\sum_{n=1}^{\infty} \frac{2}{n^2+4}$ b) $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$

Integral Test
Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$, then if $\int_1^\infty f(x)dx$ is convergent then $\sum a_n$ is also convergent, and if $\int_1^\infty f(x)dx$ is divergent then so is $\sum a_n$.

$$\lim_{t \to \infty} \int_1^t \frac{2}{x^2+4} dx$$

$$2 \cdot \frac{1}{2} \tan^{-1} \frac{x}{2} \Big|_1^t$$

$$\lim_{t \to \infty} \left[\left(\tan^{-1} \frac{t}{2}\right) - \tan^{-1}(1/2) \right]$$

$$\pi/2 - \tan^{-1}(1/2) \text{ so convergent}$$

$$\lim_{t \to \infty} \int_1^t \frac{1}{\ln x} \cdot \frac{1}{x} dx$$

$$\lim_{t \to \infty} \ln(\ln x) \Big|_1^t$$

$$\lim_{t \to \infty} \left(\ln(\ln t) - \ln(\ln 1) \right)$$

∞ $\ln(0)$ DNE

So divergent

8. State and use the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{n+2}{(n+1)^3}$ converges or diverges.

Limit Comparison Test

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.
If $\lim_{n \to \infty} \frac{a_n}{b_n} = c$, where c is a finite number, $c > 0$, then either both series converge, or both diverge.

$b_n = \frac{n+2}{(n+1)^3}$ choose $a_n = \frac{1}{n^2}$ which is convergent by p-series, $p > 1$

$$\lim_{n \to \infty} \left(\frac{\frac{1}{n^2}}{\frac{n+2}{(n+1)^3}} \right) = \lim_{n \to \infty} \left(\frac{1}{n^2} \cdot \frac{(n+1)^3}{n+2} \right) =$$

$$\lim_{n \to \infty} \left(\frac{n^3 + \cdots}{n^3 + \cdots} \right) = 1$$ which is a finite number > 0

so both are convergent

9. State and use The Comparison Test to determine if the following series are convergent or divergent. $\sum_{n=1}^{\infty} \frac{3}{2n^2+3n+5}$

The Comparison Test

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.

i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n then $\sum a_n$ is also convergent

ii) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n then $\sum a_n$ is also divergent.

choose $a_n = \frac{3}{2n^2}$ which is convergent by p-series $p>1$

$\frac{3}{2n^2+3n+5} \leq \frac{3}{2n^2}$ so both are convergent.

10. Evaluate: $\int_0^{\pi/3} \sin^3 2x \cos^3 2x \, dx$

$\int_0^{\pi/3} \sin^3 2x \cos^2 2x \cos 2x \, dx$

$(1-\sin^2 2x)$

$\int_0^{\pi/3} \sin^3 2x - \sin^5 2x \cos 2x \, dx$

$u^3 \quad u^5 \quad du$

$\frac{1}{2}\cdot\frac{1}{4}\sin^4 2x - \frac{1}{2}\cdot\frac{1}{6}\sin^6 2x \Big|_0^{\pi/3}$

$\frac{1}{8}\sin^4 2x - \frac{1}{12}\sin^6 2x \Big|_0^{\pi/3}$

$\frac{1}{8}(\sin 2\pi/3)^4 - \frac{1}{12}(\sin 2\pi/3)^6 =$

$\frac{1}{8}\left(\frac{\sqrt{3}}{2}\right)^4 - \frac{1}{12}\left(\frac{\sqrt{3}}{2}\right)^6 =$

$\frac{1}{8}\cdot\frac{9}{16} - \frac{1}{12}\cdot\frac{27}{64} = \frac{9}{128} - \frac{9}{192}$

OR $\int \sin^2 2x \cos^3 2x \quad \sin 2x \, dx$

$\int (1-\cos^2 2x)\cos^3 2x \sin 2x \, dx$

$-\frac{1}{2}\int (\cos^3 2x - \cos^5 2x) \sin 2x (-2\,dx)$

$-\frac{1}{8}\cos^4 2x + \frac{1}{12}\cos^6 2x + C \Big|_0^{\pi/3}$

$\left(-\frac{1}{8}(\cos 2\pi/3)^4 + \frac{1}{12}\left(\cos\frac{2\pi}{3}\right)^6\right) -$

$\left(-\frac{1}{8}\cos^4 0 + \frac{1}{12}\cos^6 0\right)$

$-\frac{1}{8}\left(\frac{1}{2}\right)^4 + \frac{1}{12}\left(\frac{1}{2}\right)^6 + \frac{1}{8} + \frac{1}{12}$

$-\frac{1}{8}\left(\frac{1}{16}\right) + \frac{1}{12}\left(\frac{1}{64}\right) + \frac{1}{8} + \frac{1}{12}$

$\frac{9}{256}$

11. Determine whether the sequence is convergent or divergent, monotonic or bounded. If it converges, find the limit.

a) $a_n = \frac{\sqrt{n}}{2+n}$

$\frac{1}{3}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{5}, \frac{\sqrt{4}}{6}, \frac{\sqrt{5}}{7}$

$\frac{1}{3}, \frac{1}{2}, \frac{\sqrt{3}}{5}, \frac{2}{6}, \frac{\sqrt{5}}{7}, \frac{\sqrt{6}}{8}, \frac{\sqrt{7}}{9}, \frac{\sqrt{8}}{10}, \frac{3}{11}$

non Monotonic,

Converge to 0

bounded - upper bound ½

lower bound 0

b) $a_n = \ln(n+2) - \ln(n+1)$

$a_n = \ln\left(\frac{n+2}{n+1}\right)$

monotonic, decreasing, bounded

converget

$\lim_{n\to\infty} \ln\left(\frac{n+2}{n+1}\right) = 0$

upper bound $\ln(3/2)$

lower bound 0