MATH 152
Mrs. Bonny Tighe

EXAM IA
7.1-8.3
100 points

NAME Answers

SECTION _______ Wed 3/1/06

There are 11 problems with 10 points each

1. Find dy/dx: $3 - \ln xy = 2e^{xy}$

$-\ln x - \ln y = 2e^{xy}$

$-\frac{1}{x} - \frac{1}{y}\,dy/dx = 2e^{xy}\left(x\,dy/dx + y(1)\right)$

$-y - x\,dy/dx = 2xy\,e^{xy}(x\,dy/dx + y)$

$-y - x\,dy/dx = 2x^2ye^{xy}\,dy/dx + 2xy^2e^{xy}$

$dy/dx\,(2x^2ye^{xy} + x) = -y - 2xy^2e^{xy}$

$dy/dx = \dfrac{-y - 2xy^2e^{xy}}{2x^2ye^{xy} + x}$

2. Evaluate: a) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ = ________

$2\int e^u\,du$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2e^u\Big|_1^4 = 2e^{\sqrt{x}}\Big|_1^4 = 2e^{\sqrt{4}} - 2e^{\sqrt{1}} = 2e^2 - 2e$

b) $\int \frac{3+x}{4-x^2}\,dx$ = ______

$\int \frac{3}{4-x^2}\,dx + -\frac{1}{2}\int \frac{-2x}{4-x^2}\,dx$

$\int \frac{3/4\,dx}{1-\left(\frac{x}{2}\right)^2}$

$\frac{3}{4}\tan^{-1}\left(\frac{x}{2}\right)$

$-\frac{1}{2}\int \frac{1}{u}\,du$

$-\frac{1}{2}\ln u + C$

$\frac{3}{4}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\ln|4-x^2| + C$

3. Find f '(x): a) $f(x) = \ln\sqrt{\dfrac{x^3-1}{\sin x\cos x}}$ $= \frac{1}{2}\ln(x^3-1) - \frac{1}{2}\ln\sin x - \frac{1}{2}\ln(\cos x)$

$$f'(x) = \frac{1}{2}\cdot\frac{1}{x^3-1}(3x^2) - \frac{1}{2}\cdot\frac{1}{\sin x}(\cos x) - \frac{1}{2}\cdot\frac{1}{\cos x}(-\sin x)$$

$$= \frac{3x^2}{2(x^3-1)} - \frac{1}{2}\cot x + \frac{1}{2}\tan x$$

b) $f(x) = (3-\ln\sqrt{x})^3(2^x-5^x)$

$$f'(x) = (3-\ln\sqrt{x})^3(2^x\ln 2 - 5^x\ln 5) + (2^x-5^x)\,3(3-\ln\sqrt{x})^2\left(\frac{1}{2x}\right)$$

4. Find the equation of the tangent line to the curve $y = \dfrac{\tan^{-1}x}{x^2}$ at the point $(1, \pi/4)$.

$y - y_1 = m(x - x_1)$

$y - \pi/4 = \quad (x-1)$

$$m = dy/dx = \frac{x^2\left(\frac{1}{1+x^2}\right) - \tan^{-1}x\,(2x)}{x^4}$$

at $x = 1$

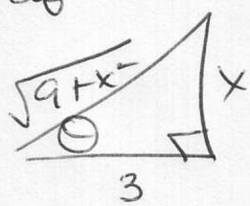
$$m = \frac{1\left(\frac{1}{2}\right) - \pi/4\,(2)}{1} = \frac{1}{2} - \pi/2$$

$y - \pi/4 = (\frac{1}{2} - \pi/2)(x-1)$

5. Integrate using trigonometric substitution: $\int \frac{\sqrt{9+x^2}}{x^2}dx$

$3\tan\theta = x$

$3\sec^2\theta\, d\theta = dx$



$\int \frac{\sqrt{9+9\tan^2\theta}}{9\tan^2\theta}(3\sec^2\theta)d\theta$

$\int \frac{\cancel{3}\sqrt{1+\tan^2\theta}}{\cancel{3}\tan^2\theta}\sec^2\theta d\theta = \int \frac{\sec^3\theta}{\tan^2\theta}d\theta =$

$\int \frac{1}{\cos^3\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}d\theta = \int \underbrace{\sec\theta}_{u}\,\underbrace{\csc^2\theta\, d\theta}_{dv}$ $\cancel{(1+\cot^2\theta)d\theta}$ $= \cancel{\int \sec\theta d\theta + \int \sec\theta\cot^2\theta d\theta}$

$\cancel{du = \sec\theta\tan\theta}$ $\quad v = -\cot\theta$

$= -\sec\theta\cot\theta + \int \cot\theta\sec\theta\tan\theta d\theta$

$-\sec\theta\cot\theta + \ln|\sec\theta + \tan\theta| + C$

so $= -\frac{\sqrt{9+x^2}}{3}\left(\frac{3}{x}\right) + \ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + C$

6. Find the numerical value of each expression.

a) $\cosh^{-1}(0) \neq$ Never

b) $\cosh(\ln 2) =$ $5/4$

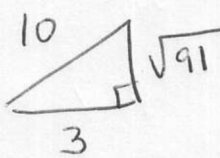
c) $\log_3 9\sqrt{3} =$ $5/2$

d) $e^{(\ln 5 - 2\ln 2)} =$ $5/4$

e) $\tan(\arccos(0.3)) =$ $\sqrt{91}/3$

$e^{\ln 5 - \ln 4}$

$e^{\ln 5/4} = 5/4$



$$\boxed{\begin{aligned} \sinh x &= \frac{e^x - e^{-x}}{2} \\ \cosh x &= \frac{e^x + e^{-x}}{2} \end{aligned}}$$

b) $\frac{e^{\ln 2} + e^{-\ln 2}}{2} = \frac{2 + \frac{1}{2}}{2} = 5/4$

a) $\frac{e^x + e^{-x}}{2} = 0$

$e^x + e^{-x} = 0$

$e^{2x} + 1 = 0 \qquad e^{2x} \neq -1$

7. Evaluate using integration by parts:

a) $\int \sec^{-1} t\, dt =$ ________

$u = \sec^{-1} t \quad dv = dt$

$du = \frac{1}{t\sqrt{t^2-1}} \quad v = t$

$= \sec^{-1} t\,(t) - \int t\left(\frac{1}{t\sqrt{t^2-1}}\right) dt =$

$t \sec^{-1} t - \int \frac{1}{\sqrt{t^2-1}}\, dt$

$t \sec^{-1} t - \cosh^{-1} t + C$

b) $\int_0^1 e^{3x} \sin 2x\, dx =$ ________

$u = e^{3x} \quad dv = \sin 2x\,dx$

$du = 3e^{3x} \quad v = -\frac{1}{2}\cos 2x$

$= -\frac{1}{2}\cos 2x\, e^{3x} - \int -\frac{1}{2}\cos 2x\,(3e^{3x})\,dx = -\frac{1}{2}\cos 2x\, e^{3x} + \frac{3}{2}\int \cos 2x\, e^{3x}\, dx$

$dv = \cos 2x \quad u = e^{3x} \quad v = \frac{1}{2}\sin 2x \quad du = 3e^{3x}$

$= -\frac{1}{2}\cos 2x\, e^{3x} + \frac{3}{2}\left(\left[e^{3x}\left(\frac{1}{2}\sin 2x\right)\right] - \int \frac{1}{2}\sin 2x\; 3e^{3x}\, dx\right)$

$\frac{13}{4}\int e^{3x}\sin 2x\,dx = \left(-\frac{1}{2}\cos 2x\, e^{3x} + \frac{3}{4}e^{3x}\sin 2x\right)\left(\frac{4}{13}\right)\Big|_0^1 =$ (on back)

8. Find the following limits. Use L'Hospital's Rule where appropriate.

a) $\lim_{x\to 0^+} x^3 \ln x =$ 0

$0 \cdot \infty$

$\lim_{x\to 0^+}\left(\frac{\ln x}{\frac{1}{x^3}}\right) = \frac{\infty}{\infty} \overset{L'H}{=}$

$\lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{\frac{-3}{x^4}}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}\cdot\frac{-x^4}{3}\right) = 0$

b) $\lim_{x\to\infty}\left(\frac{x-2}{x+1}\right)^x =$ e^{-3}

$\ln y = \lim_{x\to\infty} x \ln\left(\frac{x-2}{x+1}\right) = \infty \cdot 0$

$\ln y = \lim_{x\to\infty} \frac{\ln(x-2) - \ln(x+1)}{\frac{1}{x}} \overset{L'H}{=}$

$= \lim_{x\to\infty} \frac{\frac{1}{x-2} - \frac{1}{x+1}}{-1/x^2} = \lim_{x\to\infty} \frac{x+1-(x-2)}{(x-2)(x+1)}\cdot\frac{x^2}{1}$

$\lim_{x\to\infty} \frac{-3x^2}{(x-2)(x+1)} = \lim_{x\to\infty} \frac{-3x^2}{x^2-x-2}$

$\ln y$ -3

so e^{-3}

9. Use logarithmic differentiation to find the derivative for $f(x) = \left(\frac{x-2}{x+5}\right)^{\sin x}$

$\ln f(x) = \sin x \ln\left(\frac{x-2}{x+5}\right)$

$\frac{1}{y} \, dy/dx = \sin x \left(\frac{1}{x-2} - \frac{1}{x+5}\right) + \ln\left(\frac{x-2}{x+5}\right)(\cos x)$

$$\boxed{dy/dx = \left[\sin x\left(\frac{1}{x-2} - \frac{1}{x+5}\right) + \ln\left(\frac{x-2}{x+5}\right)\cos x\right]\left(\frac{x-2}{x+5}\right)^{\sin x}}$$

10. Evaluate the integral. $\int_0^{\pi/3} \cos^3 x \sin^4 x \, dx$

$\int \underbrace{\cos^2 x}_{(1-\sin^2 x)} \cos x \sin^4 x \, dx$

$\int_0^{\pi/3} \sin^4 x \cos x \, dx - \int_0^{\pi/3} \sin^6 x \cos x \, dx$

$\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x \Big|_0^{\pi/3} =$

$\frac{1}{5}(\sin \pi/3)^5 - \frac{1}{7}(\sin \pi/3)^7 - \left[\frac{1}{5}(\sin 0)^5 - \frac{1}{7}(\sin 0)^7\right]$

$$\boxed{\frac{1}{5}\left(\frac{\sqrt{3}}{2}\right)^5 - \frac{1}{7}\left(\frac{\sqrt{3}}{2}\right)^7}$$

11. Evaluate: a) $\int \tan^3 x \sec^5 x\, dx$ = ______________

$\int \tan^2 x \sec^4 x \,(\sec x \tan x\, dx)$

$(\sec^2 x - 1)$

$\int \sec^6 x (\sec x \tan x) dx - \int \sec^4 x \sec x \tan x\, dx$

$\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

b) $\int \frac{\csc^2(\ln x)}{x} dx$ = ______________ $\int \csc^2(\ln x) \cdot \frac{1}{x}\, dx$

$\int \csc^2 u\, du$

$-\cot u + C$

$-\cot(\ln x) + C$

$$\int e^{3x} \sin 2x \, dx$$

$u = e^{3x}$ $\quad dv = \sin 2x\,dx$

$du = 3e^{3x}$ $\qquad v = -\frac{1}{2}\cos 2x dx$

$= e^{3x}(-\frac{1}{2}\cos 2x) - \int -\frac{1}{2}\cos 2x\,(3e^{3x})dx$

$-\frac{1}{2}e^{3x}\cos 2x + \frac{3}{2}\int \cos 2x\, e^{3x}\,dx$

$dv = \cos 2x$ $\quad u = e^{3x}$ $\quad du = 3e^{3x}$

$v = \frac{1}{2}\sin 2x$

$= -\frac{1}{2}e^{3x}\cos 2x + \frac{3}{2}\left[e^{3x}\left(\frac{1}{2}\sin 2x\right) - \int \frac{1}{2}\sin 2x\, 3e^{3x}dx\right]$

$= -\frac{1}{2}e^{3x}\cos 2x + \frac{3}{4}e^{3x}\sin 2x - \frac{9}{4}\int \sin 2x\, e^{3x}dx$

$\frac{13}{4}\int e^{3x}\sin 2x\,dx = \left(-\frac{1}{2}e^{3x}\cos 2x + \frac{3}{4}e^{3x}\sin 2x\right)\frac{4}{13}$

$-\frac{2}{13}e^{3x}\cos 2x + \frac{3}{13}e^{3x}\sin 2x \Big|_0^1$

$\left(-\frac{2}{13}e^{3}\cos 2 + \frac{3}{13}e^{3}\sin 2\right) - \left(-\frac{2}{13}e^{0}\cos 0 + \frac{3}{13}e^{0}\sin 0\right)$

$-\frac{2}{13}e^{3}\cos 2 + \frac{3}{13}e^{3}\sin 2 + \frac{2}{13}$