MATH 152
Mrs. Bonny Tighe

EXAM I

7.1-8.3
100 points

NAME Answers

SECTION ______ Wed 3/1/06

There are 11 problems with 10 points each

1. Find f '(x): a) $f(x) = \ln\sqrt{\dfrac{x^3-1}{\tan x}}$ $= \frac{1}{2}\ln(x^3-1) - \frac{1}{2}\ln(\tan x)$

$f'(x) = \frac{1}{2}\left(\frac{1}{x^3-1}\right)(3x^2) - \frac{1}{2}\cdot\frac{1}{\tan x}(\sec^2 x)$

$f'(x) = \dfrac{3x^2}{2(x^3-1)} - \frac{1}{2}\cot x \sec^2 x$ $= \dfrac{3x^2}{2(x^3-1)} - \dfrac{1}{\sin 2x}$

b) $f(x) = (3-e^{\sqrt{x}})^4(3^x+5^x)$

$f'(x) = (3-e^{\sqrt{x}})^4(3^x\ln 3 + 5^x\ln 5) + (3^x+5^x)(\cancel{4}^{2})(3-e^{\sqrt{x}})^3(-e^{\sqrt{x}})\left(\frac{1}{\cancel{2}\sqrt{x}}\right)$

$(3-e^{\sqrt{x}})^3\left[(3-e^{\sqrt{x}})(3^x\ln 3+5^x\ln 5) + 2(-e^{\sqrt{x}})(3^x+5^x)\right]$

2. Find the equation of the tangent line to the curve $y = \dfrac{\sin^{-1}x}{x^2}$ at the point $\cancel{(1, \pi/2)}$ $\left(\frac{1}{2}, \frac{2\pi}{3}\right)$

$y - y_1 = m(x - x_1)$

$m = dy/dx = \dfrac{x^4\left(\frac{1}{\sqrt{1-x^2}}\right) - \sin^{-1}x\,(2x)}{x^4}$

$y - 2\pi/3 = \left(\frac{8}{\sqrt{3}} - \frac{8\pi}{3}\right)(x - 1/2)$

$y - 2\pi/3 = \left(\frac{8}{\sqrt{3}} - \frac{8\pi}{3}\right)(x - 1/2)$

at $x = 1/2$

$\dfrac{\left(\frac{1}{2}\right)^2\left(\frac{1}{\sqrt{1-1/4}}\right) - (\sin^{-1}(\frac{1}{2})(1))}{(1/2)^4\,(1/4)}$

$\left(\frac{4}{1}\right)\frac{1}{\sqrt{3/4}} - \left(\frac{\pi}{6}\right)\left(\frac{16}{1}\right)$

$\dfrac{8}{\sqrt{3}} - \dfrac{8\pi}{3}$

#4 or $3\int \frac{1-\sin^2\theta}{\sin\theta} = 3\int \csc\theta - \sin\theta \, d\theta$

3. Evaluate: a) $\int_0^1 x\, e^{-x^2}\, dx =$ ________

$3\ln|\csc\theta - \cot\theta| + \cos\theta + C$

$u = -x^2$

$du = -2x$

$-\frac{1}{2}\int e^u\, du = -\frac{1}{2} e^u \Big|_0^1 =$

$-\frac{1}{2} e^{-x^2}\Big|_0^1 = -\frac{1}{2}(e^{-1}) - \left(-\frac{1}{2}\right)e^0 = -\frac{1}{2}e^{-1} + \frac{1}{2}$

b) $\int \frac{2-x}{3-x^2}dx =$ ______

$\int \frac{\cancel{2}/3}{\cancel{3}/3 - x^2/3} + \frac{1}{2}\int \frac{-2x}{3-x^2}\, dx$

$\cancel{2\int \frac{1}{3-x^2}dx}$

$\frac{2}{\sqrt{3}}$

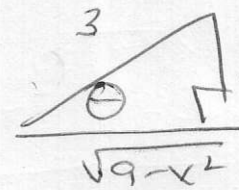
$\frac{2}{3}\int \frac{1\, dx}{1-\left(\frac{x}{\sqrt{3}}\right)^2}$ $\quad \frac{1}{2}\int \frac{1}{u}\, du$

$\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{1}{2}\ln|3-x^2| + C$

4. Integrate using trigonometric substitution: $\int \frac{\sqrt{9-x^2}}{x}dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$



$\int \frac{\sqrt{9-9\sin^2\theta}}{3\sin\theta}\, 3\cos\theta\, d\theta$

$\int \frac{\sqrt{9(1-\sin^2\theta)}\ \cancel{3}\cos\theta\, d\theta}{\cancel{3}\sin\theta}$

$\int \frac{3\cos^2\theta}{\sin\theta}\, d\theta = 3\int \frac{\cos\theta}{\sin\theta}\cdot \cos\theta\, d\theta = 3\int \underbrace{\cot\theta}_{u}\ \underbrace{\cos\theta\, d\theta}_{dv}$

$v = \sin\theta$

$du = -\csc^2\theta\, d\theta$

$3\left[\cot\theta\sin\theta + \int \sin\theta\csc^2\theta\, d\theta\right] = 3\cot\theta\sin\theta + 3\int \csc\theta\, d\theta$

$3\cot\theta\sin\theta + 3\ln|\csc\theta - \cot\theta| + C =$

$3\left(\frac{\sqrt{9-x^2}}{x}\right)\left(\frac{x}{3}\right) + 3\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + C = \sqrt{9-x^2} + 3\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + C$

5. Find dy/dx: $\ln xy = 2 - e^{xy}$ $\quad \ln x + \ln y = 2 - e^{xy}$

$\frac{1}{x} + \frac{1}{y}\, dy/dx = -e^{xy}(x\, dy/dx + y(1)) =$

$y + x\, dy/dx = -xye^{xy}(x\, dy/dx + y) =$

$y + x\, dy/dx = -x^2ye^{xy}\, dy/dx - xy^2e^{xy}$

$dy/dx(x + x^2ye^{xy}) = -y - xy^2e^{xy}$

$$dy/dx = \frac{-y - xy^2e^{xy}}{x + x^2ye^{xy}}$$

6. Evaluate using integration by parts:

a) $\int \tan^{-1} t\, dt =$ ________ $= t\cdot\tan^{-1}t - \int \tan^{-1} t\ (dt)$

v du

$dv = \frac{1}{1+t^2}$ $\quad u = t$ $\qquad t\tan^{-1}t - t$

$$t\tan^{-1}t - \frac{1}{2}\ln(1+t^2) + C$$

b) $\int_0^1 e^{2x}\cos 3x\, dx =$ ________ $= e^{2x}\left(\frac{1}{3}\sin 3x\right) - \int \frac{1}{3}\sin 3x\,(2e^{2x})\,dx$

u dv

$du = 2e^{2x}$ $\quad v = \frac{1}{3}\sin 3x$ $\qquad \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int \sin 3x\, e^{2x}\, dx$

dv u

$du = 2e^{2x}$

$v = -\frac{1}{3}\cos 3x$

$\int_0^1 e^{2x}\cos 3x\, dx = \frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\left[2e^{2x}\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x\,(2e^{2x})\,dx\right.$

$\frac{1}{2}e^{2x}\sin 3x + \frac{3}{4}e^{2x}\cos 3x - \frac{9}{4}\int \cos 3x\, e^{2x}\, dx$

$\frac{13}{4}\int_0^1 e^{2x}\cos 3x\, dx = \left(\frac{1}{2}e^{2x}\sin 3x + \frac{3}{4}e^{2x}\cos 3x\right)\left(\frac{4}{13}\right)\Big|_0^1$

$\left(\frac{2}{13}e^2\sin 3 + \frac{3}{13}e^2\cos 3\right) - \left(\frac{2}{13}(1)(0) + \frac{3}{13}(1)(1)\right) = \frac{2}{13}e^2\sin 3 + \frac{3}{13}e^2\cos 3 - \frac{3}{13}$

7. Find the following limits. Use L'Hospital's Rule where appropriate.

a) $\lim_{x\to 0^+} x^2 \ln x =$ $\underline{0}$

b) $\lim_{x\to\infty}\left(\frac{x-2}{2x+1}\right)^x = \underline{e^{-5/2}}$

$0\cdot\infty$

$\lim_{x\to 0^+}\left(\frac{\ln x}{\frac{1}{x^2}}\right) = \frac{\infty}{\infty}$ = L'H

$\lim_{x\to 0^+} = \left(\frac{\frac{1}{x}}{-2/x^3}\right) =$

$\lim_{x\to 0^+} = \frac{1}{x}\left(\frac{-x^3}{2}\right) = 0$

$\ln y = \lim_{x\to\infty} x\ln\left(\frac{x-2}{2x+1}\right) =$

$\lim_{x\to\infty}\left(\frac{\ln(x-2)-\ln(2x+1)}{\frac{1}{x}}\right) =$ L'H

$\lim_{x\to\infty}\left(\frac{\frac{1}{x-2}-\frac{2}{2x+1}}{-1/x^2}\right) = \lim_{x\to\infty}\frac{2x+1-2(x-2)}{(x-2)(2x+1)}$

$\lim_{x\to\infty}\left(\frac{5}{2x^2-3x-2}(-x^2)\right) = -5/2 \quad e^{-5/2} = e^{-5/2}$

8. Find the numerical value of each expression.

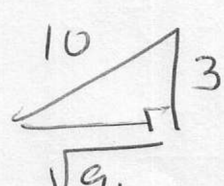
a) $\cosh^{-1}(0) =$ $\underline{\text{never}}$

b) $\cosh(\ln 2) =$ $\underline{5/4}$

c) $\log_2 8\sqrt{2} =$ $\underline{7/2}$

d) $e^{(\ln 5 - 2\ln 2)} =$ $\underline{5/4}$

e) $\tan(\arcsin(0.3)) =$ $\underline{3/\sqrt{91}}$



a) $\cosh x = \frac{e^x+e^{-x}}{2}$ $\quad \frac{e^x+e^{-x}}{2} \neq 0$ never

$\sinh x = \frac{e^x-e^{-x}}{2}$

b) $\frac{e^{\ln 2}+e^{-\ln 2}}{2} = \frac{2+1/2}{2}$

$1+\frac{1}{4} = 5/4$

d) $e^{\ln(5/4)} = 5/4$

9. Use logarithmic differentiation to find the derivative for $f(x)=\left(\dfrac{x+2}{x-3}\right)^{x}$

$\ln y = x\left[\ln(x+2)-\ln(x-3)\right]$

$\frac{1}{y}\,dy/dx = x\left[\frac{1}{x+2}-\frac{1}{x-3}\right] + \ln\left(\frac{x+2}{x-3}\right)(1)$

$dy/dx = \left[x\left(\frac{1}{x+2}-\frac{1}{x-3}\right) + \ln\left(\frac{x+2}{x-3}\right)\right]\left(\frac{x+2}{x-3}\right)^{x}$

10. Evaluate the integral. $\int_0^{\pi/3} \tan^5 x \sec^4 x\, dx$

or

$\int \sec^3 x \tan^4 x \sec x \tan x\, dx$

$\int \sec^3 x\,(\sec^2 x - 1)^2 \sec x \tan x\, dx$

$\int (\sec^7 x - 2\sec^5 x + \sec^3 x)$ " "

$\frac{1}{8}\sec^8 x - \frac{2}{6}\sec^6 x + \frac{1}{4}\sec^4 x \Big|_0^{\pi/3}$

$\int_0^{\pi/3} \tan^5 x \sec^2 x\,(1+\tan^2 x)\,dx =$

$\int_0^{\pi/3} \tan^5 x \sec^2 x\,dx + \int_0^{\pi/3} \tan^7 x \sec^2 x\,dx$

$\frac{1}{6}\tan^6 x + \frac{1}{8}\tan^8 x \Big|_0^{\pi/3}$

$\frac{1}{6}(\tan(\pi/3))^6 + \frac{1}{8}(\tan \pi/3)^8 - \left(\frac{1}{6}\tan^6 0 + \frac{1}{8}\tan^8 0\right)$

$\frac{1}{6}(\sqrt{3})^6 + \frac{1}{8}(\sqrt{3})^8 - 0$

$\frac{1}{6}(27) + \frac{1}{8}(81)$

$\frac{27}{6} + \frac{81}{8}$

$\left(\frac{4}{4}\right)\frac{9}{2} + \frac{81}{8}$

$\frac{36+81}{8} = \frac{117}{8} = 14\frac{5}{8}$

$\sin x \sin^2 x$

11. Evaluate: a) $\int \sin^3 x \cos^2 x\, dx$ = ____________

$\int \sin x (1-\cos^2 x) \cos^2 x\, dx$

$u = \cos x$

$du = -\sin x\, dx$

$\int \sin x \cos^2 x\, dx - \int \cos^4 x \sin x\, dx$

$-\int u^2\, du + \int u^4\, du$

$-\frac{1}{3}u^3 + \frac{1}{5}u^5 + C = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

b) $\int \frac{\sec^2(\ln x)}{x} dx$ = ______________

$\int \sec^2(\ln x)\, \frac{1}{x}\, dx$

$\int \sec^2 u\, du$

$\tan u + C = \tan(\ln x) + C$