MATH 151
Mrs. Bonny Tighe

QUIZ 8A
25 points
5.1,5.2

NAME Answers

Section ________ 4/21/06

1. Find an expression for the area under the graph of $f(x)=2\sin^3 3x + x\sqrt{x}$ on the interval [1,5] as a Riemann Sums using summation notation. Do not evaluate.

$\Delta x = \frac{5-1}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\left[2\sin^3 3\left(\frac{4i}{n}+1\right) + \left(\frac{4i}{n}+1\right)\sqrt{1+\frac{4i}{n}}\right]$$

2. Determine a region whose area is equal to each of the following and express each as a definite integral, but do not evaluate.

a) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{8n}\sqrt{\frac{\pi^2 i^2}{64n^2}+\sin\frac{\pi i}{8n}}$

$$\int_0^{\pi/8} \sqrt{x^2+\sin x}\, dx$$

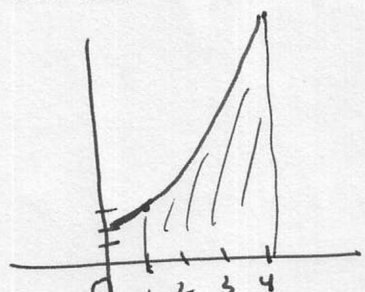
b) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\left(\sqrt{\frac{6i}{n}}+\frac{n}{3i}-\tan\frac{3i}{n}\right)$

$$\int_0^{3} \left(\sqrt{2x} + \frac{1}{x} - \tan x\right) dx$$

3. Estimate the area under the graph $f(x) = x^2 + 2$ from $x = 0$ to $x = 4$ using $n=4$, four approximating rectangles and the right endpoints, the left endpoints and then the midpoints. Graph $f(x)$. Which estimation is an underestimate or an overestimate?

$A_R = 1(f(1)) + 1f(2) + 1f(3) + 1f(4) =$

$1^2+2 + 4+2 + 9+2 + 16+2 = 38$ too big overestimate



$A_L = 1f(0) + f(1) + f(2) + f(3) =$

$2 + 3 + 6 + 11 = 22$ too small

$\Delta x = \frac{4-0}{4} = 1$

$A_m = 1f(\frac{1}{2}) + f(\frac{3}{2}) + f(\frac{5}{2}) + f(\frac{7}{2}) =$

$\frac{1}{4}+2 + \frac{9}{4}+2 + \frac{25}{4}+2 + \frac{49}{4}+2$

$8 + \frac{84}{4} = 8 + 21 = 29$ best estimate

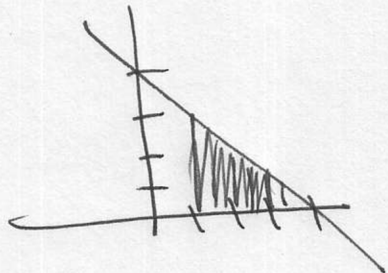
$\begin{array}{r} 35 \\ 49 \\ \hline 84 \end{array}$

4. Given that $\int_2^5 f(x)dx = 3/8$ *and* $\int_{-2}^{2} f(x)dx = 1/8$, *what is* $\int_{-2}^{5} f(t)dt$?

$4/8$ or $1/2$

5. Evaluate the integral by interpreting it in terms of areas, the graph and using elementary geometry. $\int_1^4 (4-x)dx$

$A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 9/2$ $\cancel{4}$



6. Evaluate the definite integral $\int_1^3 (2+3x-x^2)dx$ using the definition (the limit of the summation) $\sum_{i=1}^{n} c = cn$ $\quad \sum_{i=1}^{n} i = \frac{n^2}{2}+\frac{n}{2}$ $\quad \sum_{i=1}^{n} i^2 = \frac{2n^3}{6}+\frac{3n^2}{6}+\frac{n}{6}$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(2+3\left(1+\frac{2i}{n}\right)-\left(1+\frac{2i}{n}\right)^2\right) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(2+3+\frac{6i}{n}-1-\frac{4i}{n}-\frac{4i^2}{n^2}\right)$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(4+\frac{2i}{n}-\frac{4i^2}{n^2}\right) = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{8}{n}+\frac{4i}{n^2}-\frac{8i^2}{n^3}\right)$$ (Substitute from above)

$$\lim_{n\to\infty}\left(\frac{8}{n}(n)+\frac{4}{n^2}\left(\frac{n^2}{2}+\frac{n}{2}\right)-\frac{8}{n^3}\left(\frac{2n^3}{6}+\frac{3n^2}{6}+\frac{n}{6}\right)\right)$$

$$\lim_{n\to\infty}\left(8+2+\frac{2}{n}-\frac{16}{6}-\frac{24}{6n}-\frac{8}{6n^2}\right) = 10-\frac{8}{3} = \frac{22}{3} = 7\frac{1}{3}$$

Check $\int_1^3 (2+3x-x^2)\,dx = 2x+\frac{3}{2}x^2-\frac{1}{3}x^3\Big|_1^3 =$

$\left(6+\frac{3}{2}(9)-\frac{1}{3}(27)\right)-\left(2+\frac{3}{2}-\frac{1}{3}\right)$

$6+\frac{27}{2}-9-2-\frac{3}{2}+\frac{1}{3} = -5+12+\frac{1}{3} = 7\frac{1}{3}$ ✓