MATH 151
Mrs. Bonny Tighe

QUIZ 8
25 points
5.1,5.2

NAME Answers

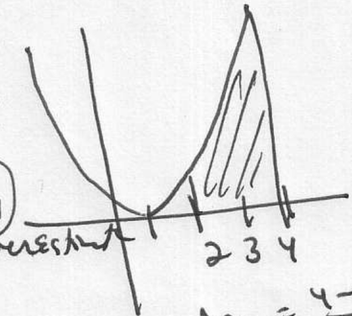
Section ________ 4/21/06

1. Estimate the area under the graph $f(x) = (x-1)^2$ from x = 2 to x = 4 using four, n = 4, approximating rectangles and right endpoints, left endpoints and midpoints. Graph f(x). Which estimation is an underestimate or an overestimate?

$A_R = \frac{1}{2}f(5/2) + \frac{1}{2}f(3) + \frac{1}{2}f(\frac{7}{2}) + \frac{1}{2}f(4)$

$\frac{1}{2}\left[\frac{9}{4} + 4 + \frac{25}{4} + 9\right] = \frac{1}{2}\left[13 + \frac{34}{4}\right] = \frac{13}{2} + \frac{17}{4} = \frac{43}{4}$

$10\frac{3}{4}$ overestimate

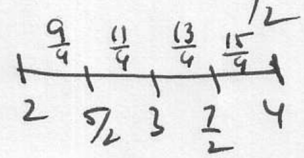


$\Delta x = \frac{4-2}{4} = \frac{1}{2}$

$A_L = \frac{1}{2}f(2) + \frac{1}{2}f(5/2) + \frac{1}{2}f(3) + \frac{1}{2}f(7/2)$

$\frac{1}{2}\left[1 + \frac{9}{4} + 4 + \frac{25}{4}\right] = \frac{1}{2}\left[5 + \frac{34}{4}\right] = \frac{5}{2} + \frac{17}{4}$

$\frac{27}{4} = 6\frac{3}{4}$ underestimate



$A_M = \frac{1}{2}f(\frac{9}{4}) + \frac{1}{2}f(\frac{11}{4}) + \frac{1}{2}f(\frac{13}{4}) + \frac{1}{2}f(\frac{15}{4})$

$\frac{1}{2}\left(\frac{25}{16}\right) + \frac{1}{2}\left(\frac{49}{16}\right) + \frac{1}{2}\left(\frac{81}{16}\right) + \frac{1}{2}\left(\frac{121}{16}\right) =$

$\frac{1}{2}\left[\frac{276}{16}\right] = \frac{138}{16} = \frac{69}{8} = 8\frac{5}{8}$ closest estimate

2. Find an expression for the area under the graph of $f(x) = \cos^3 x^2 - x\sqrt{x}$ on the interval [1,4] as a Riemann Sums using summation notation. Do not evaluate.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[\cos^3\left(\frac{3i}{n}+1\right)^2 - \left(\frac{3i}{n}+1\right)\sqrt{\left(\frac{3i}{n}+1\right)}\right]$$

3. Determine a region whose area is equal to each of the following and express each as a **definite integral**, but do not evaluate.

a) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{2n}\sqrt{3\frac{i\pi}{2n} + \tan\frac{i\pi}{2n}}$

$$\int_0^{\pi/2} \sqrt{3x + \tan x}\, dx$$

b) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{5}{n}\left(\sqrt{\frac{10i}{n}} + \frac{n}{5i} - \sin\frac{5i}{n}\right)$

$$\int_0^5 \left(\sqrt{2x} + \frac{1}{x} - \sin x\right) dx$$

4. Given that $\int_{2}^{5} f(x)dx = 2/7$ *and* $\int_{-2}^{2} f(x)dx = 4/7$, *what is* $\int_{-2}^{5} f(t)dt$?

$6/7$

5. Evaluate the definite integral $\int_{2}^{4} (x^2 + x - 1)dx$ using the definition (the limit of the summation) $\sum_{i=1}^{n} c = cn$ $\quad \sum_{i=1}^{n} i = \frac{n^2}{2} + \frac{n}{2}$ $\quad \sum_{i=1}^{n} i^2 = \frac{2n^3}{6} + \frac{3n^2}{6} + \frac{n}{6}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[\left(2+\frac{2i}{n}\right)^2 + 2+\frac{2i}{n} - 1\right] = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[4+\frac{8i}{n}+\frac{4i^2}{n^2}+2+\frac{2i}{n}-1\right]$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[5+\frac{10i}{n}+\frac{4i^2}{n^2}\right] = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{10}{n}+\frac{20i}{n^2}+\frac{8i^2}{n^3}\right)$$

now substitute to remove Σ

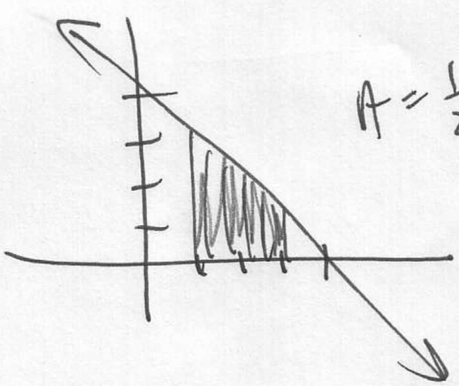
$$\lim_{n\to\infty} \left[\frac{10}{n}(n) + \frac{20}{n^2}\left(\frac{n^2}{2}+\frac{n}{2}\right) + \frac{8}{n^3}\left(\frac{2n^3}{6}+\frac{3n^2}{6}+\frac{n}{6}\right)\right]$$

$$\lim_{n\to\infty} \left[10 + 10 + \frac{20}{2n} + \frac{16}{6} + \frac{24}{6n} + \frac{8}{6n^2}\right] = 20 + \frac{16}{6} = 20 + \frac{8}{3} = 22\frac{2}{3}$$

Check $\frac{1}{3}x^3 + \frac{1}{2}x^2 - x \Big|_2^4 = \left(\frac{1}{3}(64) + \frac{1}{2}(16) - 4\right) - \left(\frac{1}{3}(8) + \frac{1}{2}(4) - 2\right)$

$\frac{64}{3} + 8 - 4 - \frac{8}{3} - \cancel{2} + \cancel{2} = \frac{64}{3} - \frac{8}{3} + 4 = \frac{68}{3} = 22\frac{2}{3}$ ✓

6. Evaluate the integral by interpreting it in terms of areas using graphing and simple geometry. $\int^{3} (4 - x)dx$



$A = \frac{1}{2}bh$

$A = \frac{1}{2}(3)(3)$ $\frac{9}{2} - \frac{1}{2}$ $A = 4$