MATH 151 **QUIZ 6A** NAME Answers

Mrs. Bonny Tighe 25 points

4.4,4.5,4.7 Section _____ Fri 3/31/06

1. Find the limit:

a) $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x+2)(2-x)} =$ 2

b) $\lim_{x\to\infty} x - \sqrt{x^2+3x+2} =$ $-3/2$

di $x\to\infty$ $\frac{-2x^2+5x-3}{-x^2\ldots}$

$\frac{x-\sqrt{x^2+3x+2}}{1} \cdot \frac{\sqrt{x^2+3x+2}+x}{x+\sqrt{x^2+3x+2}} =$

$\frac{x^2-(x^2+3x+2)}{x+\sqrt{x^2+3x+2}} = \frac{-3x-2}{x+\sqrt{x^2+3x+2}}$

c) $\lim_{x\to\infty} \frac{x^2-3}{\sqrt{x^3+2}} =$ $+\infty$

$\frac{x^2}{x^{3/2}}$

d) $\lim_{x\to\infty} (x-1)^4(2-x)^3(x+2)^2 =$ $-\infty$

$-x^9$

3. Show that any third-degree polynomial, $f(x) = ax^3+bx^2+cx+d$, always has exactly one point of inflection.

↑ $a \neq 0$ by definition

$f'(x) = 3ax^2+2bx+c$

$f''(x) = 6ax+2b$ which can only have one solution and no undefineds so there is one and only one inflection point

4. Find the vertical asymptotes and the slant asymptotes for the following functions.

a) $f(x) = \frac{2x^2+3}{x-1}$

VA: $x=1$

HA:

Slant $y = 2x+2$

$$x-1 \overline{)\,2x^2+3}\quad 2x+2$$
$$-(2x^2-2x)$$
$$2x+3$$

b) $g(x) = \frac{2+3x-4x^2+4x^3}{2x^2-8}$

VA $= x = \pm 2$

Slant $= y = 2x-2$

$$2x^2-8 \overline{)\,4x^3-4x^2+3x+2}\quad 2x-2$$
$$-(4x^3-0x^2-16x)$$
$$-4x^2+19x$$
$$-(-4x^2+16)$$
$$19x-14$$

$2x^2(x-3) - 6(3x-1)$

5. Sketch the graph of each of the following functions by finding the critical points, intervals of increasing and decreasing, inflection points, intervals of concave up and concave down, asymptotes and intercepts.

a) $f(x)=2x^3-6x^2-18x+6$

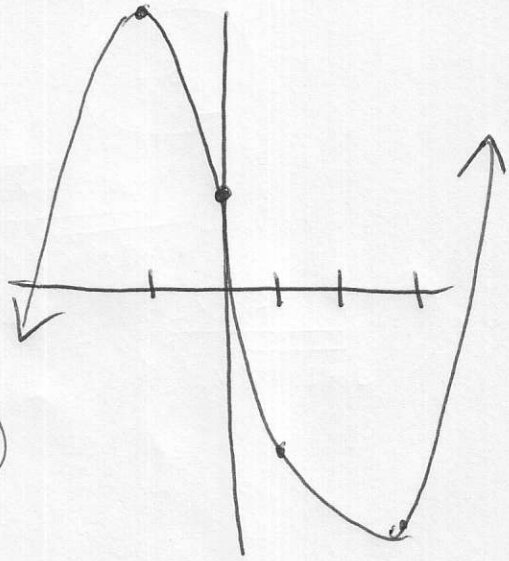
x-int:
y-int: (0,6)
no asymptotes

$f'(x) = 6x^2-12x-18 = 0$

$6(x^2-2x-3)$

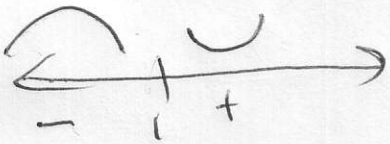
$(x-3)(x+1)=0$

$x=-1, 3$ critical #'s



$f''(x) = 12x-12 = 0$

$x=1$ Inflection pt



$f(1) = -16$ $f(3) = -21$

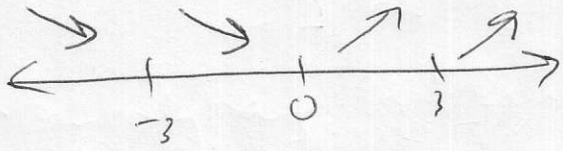
$f(-1) = 16$

b) $y = \dfrac{x^2-1}{9-x^2}$

x-int: $(\pm 1, 0)$
y-int: $(0, -1/9)$
VA: $x = \pm 3$
HA: $y = -1$

$\dfrac{dy}{dx} = \dfrac{(9-x^2)(2x)-(x^2-1)(-2x)}{(9-x^2)^2} = \dfrac{16x}{(9-x^2)^2} = 0$

$x = 0$
$x \neq \pm 3$
critical #'s



$\dfrac{d^2y}{dx^2} = \dfrac{(9-x^2)^2(16) - 16x(2)(9-x^2)^1(-2x)}{(9-x^2)^4}$

$= \dfrac{16(9-x^2)\left[(9-x^2) + x^2\right]}{(9-x^2)^4} = \dfrac{16(9+3x^2)}{(9-x^2)^3} \neq 0$

inflection pts $x \neq \pm 3$