$x^2(x-3)-9(x-3)$

$(x-3)(x^2-9) = 0$

MATH 151 **QUIZ 6** NAME Answers

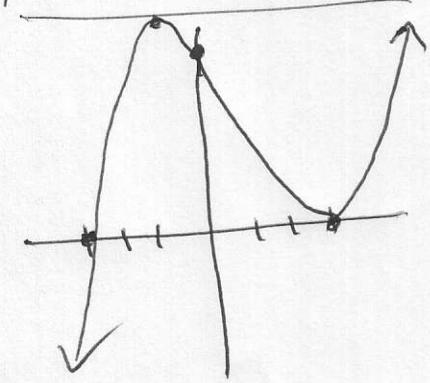
Mrs. Bonny Tighe 25 points

4.4,4.5 Section ________ Fri 3/31/06

1. Sketch the graph of each of the following functions by finding the critical points, intervals of increasing and decreasing, inflection points, intervals of concave up and concave down, asymptotes and intercepts.

a) $f(x)=x^3-3x^2-9x+27$

x-int: $(\pm 3, 0)$
y-int: $(0, 27)$
no asymptotes

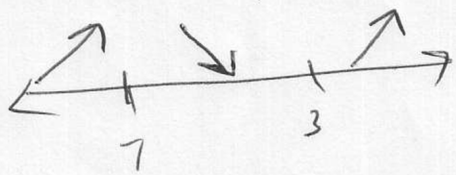


$f'(x) = 3x^2-6x-9$

$3(x^2-2x-3) = 0$

$(x-3)(x+1)$

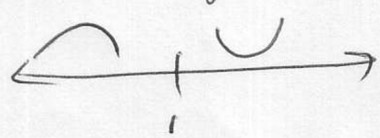
critical #s, $x = 3, -1$



$f''(x) = 6x-6 = 0$

$x = 1$

inflect-point



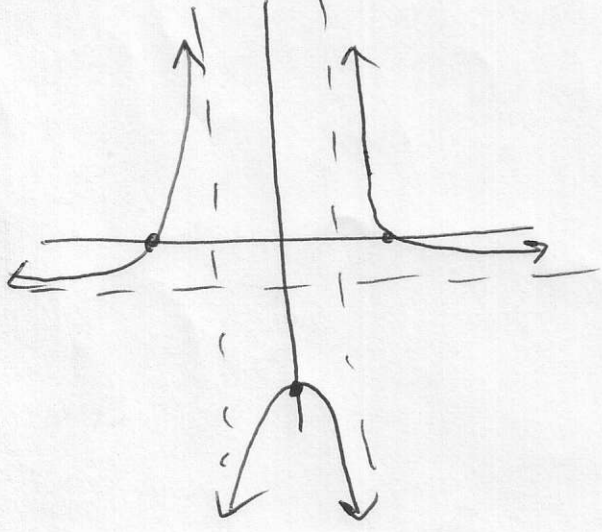
$f(-1) = -1-3+9+27 = 32$

$f(1) = 1-3-9+27 = 16$

$f(3) = 27-27-27+27 = 0$

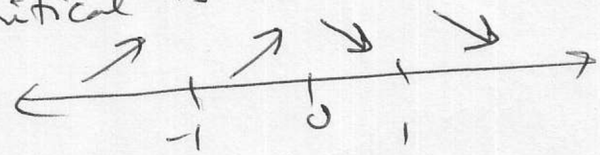
b) $y = \dfrac{x^2-4}{1-x^2}$

x-int: $(\pm 2, 0)$
y-int: $(0, -4)$
VA: $x = \pm 1$
HA: $y = -1$



$dy/dx = \dfrac{(1-x^2)(2x)-(x^2-4)(-2x)}{(1-x^2)^2} = \dfrac{-6x}{(1-x^2)^2} = 0$

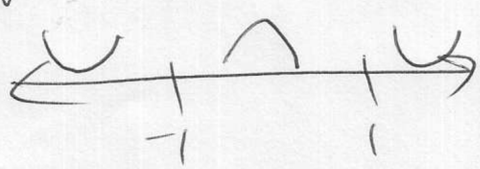
critical #s at $x = 0$, $x \neq \pm 1$



$\dfrac{d^2y}{dx^2} = \dfrac{(1-x^2)^2(-6)-(-6x)(2)(1-x^2)'(-2x)}{(1-x^2)^4}$

$= \dfrac{-6\cancel{(1-x^2)}\left[(1-x^2)+4x^2\right]}{(1-x^2)^{\cancel{4}3}} = \dfrac{-6(3x^2+1)}{(1-x^2)^3} \neq 0$

inflection-points $x \neq \pm 1$



2. Find the limit:

a) $\lim\limits_{x\to\infty} \dfrac{(x-1)(3-2x)}{(x+2)(2-3x)} =$ $2/3$

$\lim\limits_{x\to\infty} \left(\dfrac{3x - 2x^2 - 3 + 2x}{2x - 3x^2 + 4 - 6x} \right) =$

$\lim\limits_{x\to\infty} \dfrac{-2x^2 + 5x - 3}{-3x^2 - 4x + 4}$

b) $\lim\limits_{x\to+\infty} \left(\sqrt{x^2+2x-1} - x \right) =$ 1

$\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2+2-1} - x}{1} \left(\dfrac{\sqrt{x^2+2x-1} + x}{\sqrt{x^2+2x-1} + x} \right)$

$\lim\limits_{x\to\infty} \dfrac{\cancel{x^2} + 2x - 1 - \cancel{x^2}}{\sqrt{x^2+2x-1} + x} = \dfrac{2}{1+1} = 1$

c) $\lim\limits_{x\to\infty} \dfrac{2+x^2}{\sqrt{x^3+1}} =$ $+\infty$

$\lim\limits_{x\to\infty} \left(\dfrac{x^2 \ldots}{x^{3/2} \ldots} \right)$

d) $\lim\limits_{x\to\infty} (x-1)^4(2-x)^2(x+2)^2 =$ $+\infty$

$\lim\limits_{x\to\infty} \left(+x^8 \ldots \right)$

3. Show that a third-degree polynomial, $f(x) = ax^3 + bx^2 + cx + d$, always has exactly one point of inflection.

$a \neq 0$ by definition, $f'(x) = 3ax^2 + 2bx + c$, $f''(x) = 6ax + 2b$ so there can be one and only one root/or inflection point. Cannot be undefined, only a linear equation left.

4. Find the slant asymptotes and the vertical asymptotes for the following functions.

a) $f(x) = \dfrac{3-x^2}{x+2}$

VA: $x = -2$

Slant: $y = -x + 2$

$$\begin{array}{r} -x + 2 \\ x+2 \overline{\big) -x^2 + 3} \\ -(-x^2 - 2x) \\ \hline 2x + 3 \end{array}$$

b) $g(x) = \dfrac{3x^3 + 2x^2 - 5x + 2}{x^2 - 1}$

VA: $x = \pm 1$

Slat: $y = 3x + 2$

$$\begin{array}{r} 3x + 2 \\ x^2 - 1 \overline{\big) 3x^3 + 2x^2 - 5x + 2} \\ -(3x^3 \quad - 3x) \\ \hline 2x^2 - 2x + 2 \\ -(2x^2 \quad - 2) \\ \hline 2x \end{array}$$