MATH 151
Mrs. Bonny Tighe

QUIZ 5A
25 points
4.1-4.3

NAME Answers

SECTION______ Fri 3/17/06

1. Find the absolute maximum and minimum values of *f* on the given intervals.

a) $f(x) = \dfrac{x}{x^2+1}$ on [0,2]

b) $f(x) = 3x^2 - 12x + 5$ on [0,3]

absolute.
$f(0) = 0$ min
$f(1) = \frac{1}{2}$ absolute max
$f(2) = \frac{2}{5}$

$f'(x) = \dfrac{(x^2+1)-(x)(2x)}{(x^2+1)^2} = 0$

$x^2+1-2x^2 = 0$

$1 = x^2 \quad x = \pm 1$

$f'(x) = 6x - 12$

$6x - 12 = 0$

$x = 2$

$f(0) = 5$
$f(3) = -4$
$f(2) = -7$

absolute max is 5
absolute min is -7

2. For what values of the constants *a* and *b* if the function f has critical points at x = 2 and x = 1. $f(x) = x^3 + ax^2 + bx + 1$.

$3x^2 + 2ax + b = 0$

$3(4) + 2a(2) + b = 0$ and $3 + 2a + b = 0$

$-(12 + 4a + b) = 0$

$-9 - 2a = 0$

$-\frac{9}{2} = a$

$3 + (-\frac{9}{2})(2) + b = 0$

$3 - 9 + b = 0$

$b = 6$

3. Find all numbers c that satisfy the conclusion of The Mean Value Theorem if $f(x) = x^3 - 3x^2 + 4x - 1$ on the interval [1, 2].

1- $f(x)$ is cont on [1,2] yes, a polynomial
2- $f(x)$ is differentiable on (1,2) yes

So there must be a number c in (1,2) so that

$f'(c) = \dfrac{f(2)-f(1)}{2-1}$

$f(2) = 8 - 12 + 8 - 1 = 3$

$f(1) = 1 - 3 + 4 - 1 = 1$

$f'(x) = 3x^2 - 6x + 4 = \dfrac{3-1}{2-1} = 2$

$3x^2 - 6x + 2 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 4(3)(2)}}{2(3)} = \dfrac{6 \pm \sqrt{12}}{6}$

So

$c = \dfrac{6 + \sqrt{12}}{6}$

4. Find the critical numbers, intervals of increasing and decreasing, inflection points, intervals of concave up and concave down and local maximums and minimums using the first and second derivative tests.

a) f(x)= $2-15x+9x^2-x^3$

$f'(x) = -15 + 18x - 3x^2 = 0$

$-3(x^2 - 6x + 5) = 0$

$(x-5)(x-1)$

critical nubus $x = 5, +1$

$\searrow$ $\nearrow$ $\searrow$

1 5

local min at $x = 1$
local max at $x = 5$

$f''(x) = 18 - 6x = 0$ $x = 3$ inflection point

$\cup$ $\cap$

3

c) $f(x) = x\sqrt{5-x}$ (Domain $x \le 5$)

$f'(x) = x \frac{1}{2}(5-x)^{-1/2}(-1) + \sqrt{5-x}(1)$

$(5-x)^{-1/2}\left[-\frac{x}{2} + 5 - x\right] = 0$

$\dfrac{-\frac{3}{2}x + 5}{\sqrt{5-x}} = 0$ $x = 10/3$

$\nearrow$ $\searrow$

$10/3$ 5

local max at $x = 10/3$

$$f''(x) = \frac{(5-x)^{1/2}(-3/2) - (-3/2x+5)\left(\frac{-1}{2\sqrt{5-x}}\right)}{5-x}$$

$$\frac{\cancel{(5-x)^{-1/2}}\left[(5-x)(-3/2) + 1/2(-3/2x+5)\right]}{(5-x)^{3/2}} = 0$$

$-15/2 + 3/2x - 3/4x + 5/2 = 0$

$8(-15/2 + 3/2x - 3/4x + 5/2) = 0$

$-60 + 12x - 6x + 20 = 0$

$6x - 40 = 0$ $x = 6\frac{2}{3}$

$20/3$

not on domain

$\frac{20}{3}$

$f''(x) = \dfrac{6x - 40}{8(5-x)^{3/2}}$

always concave down