MATH 151
Mrs. Bonny Tighe

QUIZ 5

25 points
4.1-4.3

NAME Answers

SECTION______ Fri. 3/17/06

1. Find the absolute maximum and minimum values of f on the given interval.

$f(x) = x\sqrt{4-x^2}$ on [-1,2]

$f(-1) = -1(\sqrt{3}) = -\sqrt{3}$ absolute minimum

$f(\sqrt{2}) = 2$ absolute max

$f(2) = 2\sqrt{0} = 0$

$f'(x) = x\left(\frac{-2x}{2\sqrt{4-x^2}}\right) + \sqrt{4-x^2}(1) = 0$

$(4-x^2)^{-1/2}\left[-x^2+4-x^2\right] = 0$

$4 = 2x^2 \quad x^2 = 2 \quad x = \pm\sqrt{2}$

b) $f(x) = x + \sin 2x$ on $[0, \pi]$

$f'(x) = 1 + 2\cos 2x = 0$

$\cos 2x = -1/2$

$2x = 2\pi/3$

$x = \pi/3$

$f(0) = 0$

$f(\pi) = \pi$

$f(\pi/3) = \pi/3 + \sin 2\pi/3$

$\pi/3 + \sqrt{3}/2$

absolute min ~~value~~ = 0
absolute max value = π

2. Show that the equation $f(x) = x^3 + 3x^2 - 6x - 2$ satisfies the hypotheses of the Mean Value Theorem on the interval [0,1], and find all numbers, c, which satisfy the conclusion.

$f(x)$ is continuous on $[0,1]$ – yes a polynomial
$f(x)$ is differentiable on $(0,1)$ " "

So there is a c on (0,1) so that $f'(c) = \frac{f(1)-f(0)}{1-0}$

$f(1) = 1+3-6-2$
-4

$f(0) = -2$

$f'(x) = 3x^2 + 6x - 6 = \frac{-4-(-2)}{1-0} = -2$

$3x^2 + 6x - 4 = 0$

$(3x-1)(x+7/3) = 0$

$x = 1/3, -7/3$

so $c = 1/3$

3. For what values of the constants a and b if the function has critical points at x = 1 and x = -1. $y = x^3 + ax^2 + bx + 1$.

$3x^2 + 2ax + b = 0$

$x=1 \quad 3 + 2a + b$

$x=-1 \quad 3 - 2a + b$

$6 + 2b = 0$

$b = -3$

$3 + 2a - 3 = 0$

$a = 0$

4. Find the critical numbers, intervals of increasing and decreasing, inflection points, intervals of concave up and concave down and local maximums and minimums using the first and second derivative tests.

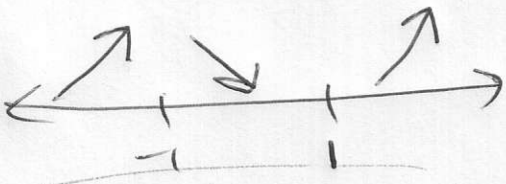
a) $f(x) = x^5 - 5x + 3$

$f'(x) = 5x^4 - 5 = 0$

$5(x^2+1)(x^2-1) = 0$

critical numbers

$x = 1, -1$



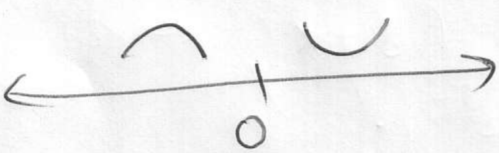
local max at $x = -1$

local min at $x = 1$

$f''(x) = 20x^3 = 0$

$x = 0$

inflection point



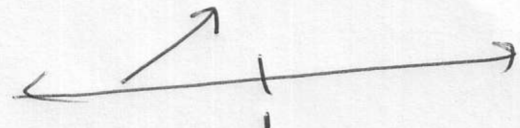
b) $f(x) = x + \sqrt{1-x}$ Doman $x \leq 1$

$f'(x) = 1 + \frac{1}{2}(1-x)^{-1/2}(-1) = 0$

$1 - \frac{1}{2\sqrt{1-x}} = 0$

$2\sqrt{1-x} = 0$

$x = 1$ critical number



$f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(1-x)^{-3/2}(-1) = 0$

$\frac{1}{4(1-x)^{3/2}} = 0$ none

no inflection points

always concave down